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THE ROLE OF MATHEMATICS IN THE DEVELOPMENT OF BIOMEDICAL ROBOTICS AND DEVICES FOR HEALTHCARE

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Abstract: Robotics is an interdisciplinary field that integrates principles from mechanical engineering, mathematics, electrical engineering, and computer science to design, construct, and operate robots. In the biomedical and healthcare fields, biomedical robots and devices have become instrumental in tasks such as surgery, rehabilitation, and diagnostics. Robotic technologies in rehabilitation assist patients in regaining movement and function following injuries or illnesses, with applications ranging from wearable devices to surgical robots. These advancements aim to enhance the precision and efficiency of healthcare services. Recent years have witnessed rapid growth in biomedical robotics, highlighting its potential to revolutionize healthcare. Mathematics plays a pivotal role in this development by providing the foundation for modelling and simulating complex biological systems, developing control algorithms, and optimizing device performance. Accurate mathematical models of systems like the musculoskeletal structure are essential for improving robotic efficiency and patient outcomes. Despite existing research on mathematical techniques in biomedical robotics, further exploration is needed to determine the most effective algorithms for various applications. This chapter examines the opportunities and challenges in applying mathematics to biomedical robots, emphasizing topics such as modelling, simulation, control, and optimization. It also explores the transformative potential of these technologies to enhance patient outcomes and reduce healthcare costs, ultimately underscoring the vital role of mathematics in shaping the future of biomedical robotics.

Keywords: Mathematics, Robotics, Biomedical and Healthcare Fields, Modelling and Simulation, Control Algorithms.

1. Introduction: The global population is rapidly growing, with most people expected to live in developing countries in the near future [1]. This demographic shift aligns with rapid technological advancements, particularly in electric vehicles, autonomous systems, and automation [2]. In healthcare, technology addresses the growing population’s needs and the shortage of healthcare professionals. Robotics and automation alleviate repetitive tasks, enhancing efficiency and enabling medical staff to focus on patient care [3]. Additionally, technology improves surgical precision and reduces invasiveness. The field of robotics began in the 1920s when Karel Čapek coined the term "robot" in his play *Rossum's Universal Robots* [4]. Today, robots are programmable machines designed to perform predefined tasks while interacting with their environments [5]. The Robot Institute of America (RIA) defines a robot as a "programmable multifunctional manipulator designed to move material, parts, tools, or specialized devices through programmed motions to perform various tasks." Robots have been used across various industries, including the military, manufacturing, space exploration, and healthcare, especially since the 1960s [6].Advancements in hardware and software have enabled robots to interact with humans, adapt to environments, and make autonomous decisions, expanding their use in manufacturing, exploration, and healthcare [7]. Biomedical robotics, which integrates robotics, mathematics, and medicine, addresses challenges like an aging population, chronic diseases, and the need for precise medical interventions [8]. The use of robotics in healthcare improves accuracy, patient outcomes, and reduces invasiveness [9]. Robots assist in surgeries, rehabilitation, and preventive care, showcasing their transformative impact on healthcare. Robotics and Artificial Intelligence (AI) have enabled robots to serve as companions and assistants in various settings, including homes, workplaces, and educational environments [10]. The healthcare sector has greatly benefited from these advancements. The first instance of robot-assisted surgery occurred in 1985 with a robotic arm performing a brain biopsy guided by a CT scanner [11]. Since then, robotics in medicine has significantly evolved, ushering in a new era of healthcare innovation. Robots are now integral in surgical systems, laparoscopic procedures, tele-presence for remote care, and rehabilitation devices. Their applications extend to diagnostics, patient monitoring, and laboratory tasks, particularly in complex fields like neurosurgery, cardiac surgery, orthopaedics, and urology. Robotics also plays a vital role in nursing, surgery, and minor medical interventions. Mathematics is key to addressing complex medical challenges. It supports the development of mathematical models that explain biological processes, predict disease trends, and design effective treatments. These models, combined with data from genetics and protein interactions, are instrumental in drug development, personalized treatments, and efficient healthcare strategies. Statistical models, differential equations, and machine learning algorithms are vital in advancing healthcare, aiding in tumour growth modelling, cardiovascular analysis, and disease risk prediction [13]. Machine learning, grounded in mathematics, enhances diagnostic accuracy in medical imaging and pathology [14]. Network models optimize healthcare delivery and resource allocation. Mathematics plays a foundational role in modern healthcare advancements, from disease understanding and treatment optimization to resource management. It is essential for the development of biomedical robots and devices, influencing their design, simulation, control, and optimization. Mathematical models simulate interactions between robots, patients, and environments, ensuring the safety and effectiveness of medical devices. Techniques such as differential equations and kinematic modelling provide accurate simulations of robotic behaviour in clinical settings [15].Optimization methods help refine robot designs by selecting appropriate materials, shapes, and movement patterns, ensuring safety and efficiency. Control algorithms like PID and adaptive controllers regulate robotic movements in surgery and rehabilitation, improving precision and safety. Mathematical algorithms also process sensory data, enabling robots to interact with tissues and provide real-time feedback [16]. Machine learning further enhances robotic systems by supporting adaptive decision-making. In conclusion, mathematics drives the development of biomedical robotics, supporting the creation of advanced systems that enhance healthcare delivery and patient outcomes. This chapter explores the mathematical foundations of biomedical robotics, discussing its opportunities, challenges, and potential impact on healthcare.

2. Mathematical Foundations of Biomedical Robotics: Robotics is an interdisciplinary field that integrates mechanics, electronics, computer science, and mathematics, forming the foundation for advancements in biomedical technology and healthcare solutions. Mathematics is essential in robotics, providing the tools needed to design, model, and control robots [17].**Linear algebra** is one of the key mathematical principles in robotics. It deals with vectors and matrices, which are used to represent spatial relationships, position, orientation, and transformations. Vectors represent quantities like position, velocity, and acceleration, while matrices model linear transformations such as rotations. Linear algebra is foundational for describing robotic kinematics and dynamics. **Kinematics** involves studying motion without considering forces, addressing problems like forward and inverse kinematics to translate joint angles into end-effectors positions [18]. **Dynamics**, rooted in physics and differential equations, describes how robots respond to forces and torques, which is crucial for tasks such as motion control and grasping objects. **Differential equations** model system dynamics, enabling precise robot movement and interaction with the environment. They are key to designing controllers that manage robotic motion. In addition to linear algebra, differential equations, kinematics, and dynamics, **probability, statistics**, and **optimization** play important roles. Probability and statistics are used in robot perception and navigation, while optimization helps with motion planning and control [19].Together, these mathematical principles enhance robot capabilities, enabling them to perform increasingly complex and precise tasks.

Definition 2.1 Vector Space: A vector space is a set of elements following some certain rules. The elements of a vector space are known as vectors. These vectors follow certain algebraic operations such as addition, subtraction, multiplication etc. vectors can also be multiplied by scalars which are elements of the vector space filed. The vector space is always denoted by capital letters whereas the vectors are denoted by the small letters.

Definition 2.2 Linear Maps: A linear map is a mapping between two vector spaces that preserves the structure of the vector space. Let U and V be two vector spaces over the same field $\mathbb{F}$, then the map $T: U \longrightarrow V$ is said to be a linear if it holds the following relation:

$$T(\alpha u_1 + \beta u_2) = \alpha T(u_1) + \beta T(u_2), \qquad \alpha, \beta \in \mathbb{F} \text{ and} u_1, u_2 \in U$$

Definition 2.3 Matrix: A matrix is a rectangular array of numbers, symbols or expressions arranged in rows and columns. In linear algebra, a matrix is a linear map from one vector space to another. Let V_1 and V_2be two finite dimensional vector spaces over the same field with dimensions m and n respectively. Let $T: V_1 \longrightarrow V_2$ be a linear map then T can expressed by a matrix order $m \times n$ with m rows and n columns and vice versa. Therefore, T can be represented by a matrix as follows:

$$T = A = [a_{ij}]_{m\times n} \text{Where } i = 1,2,3, \dots \dots \dots m \text{ and } j = 1,2,3, \dots \dots \dots n$$

algebraic operations on matrices are defined only when the operands are conformable or have suitable dimensions or orders. Addition and subtraction are defined when the orders of the matrix are same and these operations are defined element wise. The multiplication of a matrix by a scalar is always possible and it multiplies every element of the matrix by the scalar. The product of two matrix is little complex and only possible when the number of columns of first is same as the number of rows of second.

Definition 2.4 Euclidean Space: A Euclidean space is a finite dimensional vector space over the field of real numbers with a defined inner product. A Euclidean space of n-dimension is denoted by $\mathbb{F}^n$ and called n-Euclidean space. There is only one unique Euclidean space of each dimension or all Euclidean space of a particular dimension are isomorphic to each other. Therefore, a Euclidean space of n-dimension $\mathbb{F}^n$ can be represented by the cartesian co-ordinates as the real n-space $\mathbb{R}^n$ equipped with the standard dot product. Any element $v \in \mathbb{R}^n$ is written as the n-ordered touple of real numbers as follows: $v = (a_1, a_2, a_3, \dots \dots \dots a_n) \in \mathbb{R}^n$, $a_i \in \mathbb{R}$.

Definition 2.5 Three-Dimensional Space: The real coordinate n-space of dimension n, for any natural number n is denoted by $\mathbb{R}^n$ and is the set of n-touple of real numbers. The three-dimensional real coordinate space is denoted by $\mathbb{R}^3$ and is the set of 3-toupke of real numbers. The three-dimensional real space is defined as follows: $a = (a_1, a_2, a_3) \in \mathbb{R}^3$, $a_i \in \mathbb{R}$.
This three-dimensional real coordinate space can represent the position of any object in the physical world.

2.1 Rigid Body Transformations: In robotics we study the motion of the set of particles such as the link of a robot manipulator. An object is said to be a rigid body if it is not distortable and is a collection of particles such that the distance between the two particles remains unchanged irrespective of motion and forces. Thus if p and q are two points on the rigid body then it must always satisfy the following relation:

$$\|p(t) - q(t)\| = \|p(0) - q(0)\| = \text{Constant}$$

2.1.1 Rigid Body Transformation [84], [85]**:** A mapping $f: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$is said to be rigid body transformation if it holds the following properties:

(i) The length is preserved i.e., $\|f(x) - f(y)\| = \|x - y\|$, $x, y \in \mathbb{R}^3$
(ii) The cross product is preserved i.e., $f(x \times y) = f(x) \times f(y)$

2.1.2 Rigid Body Motion: A rigid motion is a movement of an object that does not change the distances between any two points on the object. This means that a rigid motion can be thought of as a transformation that preserves the shape and size of the object. The study of robot kinematics, dynamics, and control is fundamentally concerned with the motion of rigid objects. This is because robots are typically made up of rigid links and joints, and their motion can be modelled as a series of rigid motions. In this section, we will use the tools of linear algebra to provide a description of rigid body motion. Linear algebra is a powerful mathematical tool that can be used to represent and manipulate spatial data. This makes it ideal for modelling the motion of rigid objects. **The motion of a particle in Euclidean space can be described by specifying its position at each point in time, relative to a fixed Cartesian coordinate frame. To do this, we choose a set of three perpendicular axes and represent the particle's position using a triple of numbers $(x_1, x_{1,} x_3)$ in $\mathbb{R}^3$, where each number represents the distance from the particle to the corresponding axis. The particle's trajectory is represented by a parameterized curve $X(t) = (x_1(t), x_2(t), x_3(t))$, where t denotes the time and $X(t)$ denotes the position of the particle or the object in space at time t.**

2.1.3 Rotation and Translation: A **translation** is a rigid motion where every point in an object moves the same distance in the same direction, maintaining its orientation. It can be represented by the change in the object's position from a reference point to its current location.A **rotation** is a rigid motion where at least one point in the object remains in its initial position, but the object changes its orientation. The position of the object does not change, but its orientation does. Any representation of orientation can be used to define a rotation, and vice versa. Rotation is a fundamental concept in robotics, crucial for tasks such as navigation, planning, and control.

2.1.4 Rotation Matrix: Let A be the inertial frame and B be the body frame and $X_{ab}, Y_{ab}, Z_{ab} \in \mathbb{R}^3$ are the coordinates of the principal axes of frame B relative to the frame A. A matrix of order 3 can be defined as follows:

$$R_{ab} = [X_{ab}, Y_{ab}, Z_{ab}]$$

$$R_{ab} = \begin{bmatrix} \hat{x}_a . \hat{x}_b & \hat{y}_a . \hat{x}_b & \hat{z}_a . \hat{x}_b \\ \hat{x}_a . \hat{y}_b & \hat{y}_a . \hat{y}_b & \hat{z}_a . \hat{y}_b \\ \hat{x}_a . \hat{z}_b & \hat{y}_a . \hat{y}_b & \hat{z}_a . \hat{z}_b \end{bmatrix}$$

Where, $(\hat{x}_a, \hat{y}_a, \hat{z}_a)$ and $(\hat{x}_b, \hat{y}_b, \hat{z}_b)$ are the bases vectors of the frames A and B, respectively and the matrix R_{ab} is known as the rotation matrix.

The rotation matrices of the elementary transformations of frame A about axis $x_b y_b$ and z_b through an angle θ are as follows:

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\,\theta \\ 0 & \sin\theta & cos\,\theta \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\,\theta \\ 0 & \sin\theta & cos\,\theta \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\,\theta \\ 0 & \sin\theta & cos\,\theta \end{bmatrix}$$

Definition 2.6 Euler Angles: The orientation of a coordinate frame B relative to another coordinate frame A can also be represented as a vector of three angles (α, β, γ), where $\alpha, \beta,$ and γ denotes the rotation of the coordinate frame B about z-axis, y-axis and x-axis respectively. The angles (α, β, γ), are known as the Eulers angles.

Definition 2.7 Quaternions: The orientation of a coordinate frame can also be represented by some generalised complex numbers. A quaternion is a vector quantity defined as follows:

$$Q = \varepsilon_0 + \varepsilon_1 \hat{\imath} + \varepsilon_2 \hat{\jmath} + \varepsilon_3 \hat{k}, \qquad \varepsilon_i \in \mathbb{R}, i = 0, 1, 2, 3$$

Where $\varepsilon_0, \varepsilon_1, \varepsilon_2, \varepsilon_3$ are the scalars and known as the Euler parameters, the symbols $\hat{\imath}, \hat{\jmath},$ and $\hat{k}$ are called operators. The parameter ε_0 is the scalar component of Q and $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3)$ is the vector component of Q. The operator satisfies the following relation:

$$\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}\hat{k} = -1$$
$$\hat{\imath}.\hat{\jmath} = \hat{k}, \qquad \hat{\jmath}.\hat{k} = \hat{\imath}, \qquad \hat{k}.\hat{\imath} = \hat{\jmath}$$
$$\hat{\jmath}.\hat{\imath} = -\hat{k}, \qquad \hat{k}.\hat{\jmath} = -\hat{\imath}, \qquad \hat{\imath}.\hat{k}. = -\hat{\jmath}$$

2.1.5 Chasles' Theorem: Chasles theorem is one of the most fundamental results in kinematics about the most general body displacement. This theorem has two parts.The first part of the Chasles' theorem states: "Any displacement of a body in space can be accomplished by means of translation of a designated point from its initial position to final position followed by the rotation of the whole body about that point to bring that body int its final position". The second part of the theorem states: Any displacement of a body in space can be accomplished by the means of a rotation of the body about a unique line in the space accompanied by a transition of the body parallel to that line". There is a combined statement of the Chasles' theorem given as follows: "Every rigid body motion can be realised by a rotation about an axis combined with a translation parallel to that axis."

2.2 Manipulator Kinematics: Contemporary manipulators consist of interconnected rigid links and joints, which are equipped with motors for precise motion control. The choice of joints is crucial, with common options being lower pairs, such as revolute, prismatic, and helical joints. Revolute joints enable rotational motion, prismatic joints allow linear motion, and helical joints facilitate screw motion with finite pitch. The joint configuration influences the manipulator's capabilities, with cylindrical joints offering two degrees of freedom by combining revolute and prismatic elements. Spherical joints provide unrestricted rotation, while planar joints enable translation and rotation in a plane. Helical joints exhibit screw motion. Understanding these joint types is essential for designing robotic systems that offer optimal performance and adaptability for specific tasks and environments [20].

2.2.1 Forward Kinematics: The forward kinematics problem for a serial-chain manipulator is to find the position and orientation of the end-effector relative to the base given the positions of all of the joints and the values of all of the geometric link parameters. A broader formulation of the forward kinematics problem involves determining the relative position and orientation between any two specified components, considering the manipulator's geometric configuration and the joint positions' values, matching the degrees of freedom in the system. This problem is pivotal in the development of manipulator coordination algorithms, especially as joint positions, usually sensed by joint-mounted sensors, necessitate the computation of joint axis positions concerning the fixed reference frame. Let T represent the homogeneous transformation matrix describing the end-effector's pose in the reference frame, and θ denote the vector of joint variables. The forward kinematics equation is given by $T = T_1 T_2 T_3 \dots\dots\dots\dots T_n$, where each $T_i$corresponds to the transformation matrix associated with the ith joint. The overall transformation matrix T provides the position and orientation of the end-effector relative to the base frame. This mathematical formulation enables precise modelling and control of robotic manipulators, essential for various applications in fields like manufacturing, healthcare, and beyond [21].

2.2.2 Inverse Kinematics: The inverse kinematics problem in a serial-chain manipulator involves determining the joint positions by knowing the end-effector's position and orientation in relation to the base, along with the geometric link parameters. In a broader context, the problem is expressed as finding the joint positions when provided with the relative positions and orientations of two members within a mechanism. Let T_d represent the desired end-effector transformation matrix, and θ denote the vector of joint variables. The inverse kinematics problem is to find θ such that $T(\theta) = T_d$, where $T(\theta)$ is the transformation matrix representing the end-effector pose as a function of the joint variables [89], [92], [93].

Definition 2.8 Jacobian: The Jacobian of an ordinary function $X = f(q)$ with $f: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is an $m \times n$ order matrix of partial derivatives given as follows:

$$J(q) = \frac{\partial f(q)}{\partial X} = \begin{bmatrix} \frac{\partial f_1(q)}{\partial q_1} & \frac{\partial f_1(q)}{\partial q_2} & \dots\dots & \frac{\partial f_1(q)}{\partial q_n} \\ \frac{\partial f_2(q)}{\partial q_1} & \frac{\partial f_2(q)}{\partial q_2} & \dots\dots & \frac{\partial f_2(q)}{\partial q_n} \\ \dots & \dots & \dots\dots & \dots \\ \dots & \dots & \dots\dots & \dots \\ \frac{\partial f_m(q)}{\partial q_1} & \frac{\partial f_m(q)}{\partial q_2} & \dots\dots & \frac{\partial f_1(q)}{\partial q_n} \end{bmatrix}$$

where the component of the function $f(q)$ are given by $f(q) = [f_1(q), f_2(q), \dots\dots f_m(q)]^T$ and the vector$q$ is given by $q = (q_1, q_2, q_3, \dots\dots q_n)$.

2.2.3 Differential Kinematics: Mathematically the forward and inverse kinematics in robotics defines a function between the space of cartesian positions, orientations and the space of joint positions. The Jacobian of this function can determine the velocity relationship of end effectors or manipulator and joint velocities. Finding out the relationship between the joint velocities and the linear and angular velocities of the corresponding end effectors is known as differential kinematics [22].

2.2.4 Geometric Jacobian: Let us suppose that the linear velocity of the end effector is denoted by $\dot{P}_e$ and the angular velocity is denoted by w_e. These velocities are the function of the joint velocities $\dot{q}$ and represented as follows:

$$\dot{P}_e = J_P(q)\,\dot{q} \text{and} w_e = J_O(q)\,\dot{q}$$

The vector of the joint variables is denoted by q and expressed as $(q_1, q_2\,, q_3 \dots q_n)^T$. Where P_e denotes the position vector of the end effectors, J_P and J_O are the Jacobian matrix of order $3 \times n$ relating to the contribution of joint velocity $\dot{q}$ to the end effector's linear and angular velocity $\dot{P}_e$ and w_e respectively. Thus, the equation of the differential kinematics off the end effector is given by,

$$v_e = \begin{bmatrix} \dot{P}_e \\ w_e \end{bmatrix} = J(q)\,\dot{q}$$

Where v_e is known as the body velocity, and the symbol $\dot{q}$ denotes the joint velocities. $J(q)$ is the Jacobian matrix order $6 \times n$ known as the geometric Jacobian of the manipulator and defined as follows:

$$J(q) = \begin{bmatrix} J_P(q) \\ J_O(q) \end{bmatrix}$$

2.2.5 Analytical Jacobian: The geometric Jacobian of an end effector gives the impact of each joint on end effectors linear and angular velocities. If the minimum number of parameters defines the end effectors pose, then an analytical form of the Jacobian can be obtained by the differentiation of the direct kinematic function. Let P_e be the position vector of end effector frame. Then the translational velocity of the end effector can be expressed as the time derivative of P_e as follows:

$$\dot{P}_e = \frac{\partial P_e}{\partial q}\,\dot{q} = J_P(q)\,\dot{q}$$

Let φ_e denotes the minimal representation of the orientation for the rotational velocity of the end effectors. Its time derivative $\dot{\varphi}_e$ is different from the angular velocity but if $\varphi_e(q)$ is known then it is formally correct to consider the Jacobian defined as follows:

$$\dot{\varphi}_e = \frac{\partial \varphi_e}{\partial q}\,\dot{q} = J_\varphi(q)\,\dot{q}$$

Therefore, the differential kinematics equation is defined as follows:

$$\dot{x}_e = \begin{bmatrix} \dot{P}_e \\ \dot{\varphi}_e \end{bmatrix} = \begin{bmatrix} J_P(q) \\ J_\varphi(q) \end{bmatrix}\dot{q} = J_A(q)\,\dot{q}$$

Where $J_A(q)$ is known as the analytical Jacobian.

2.3 Manipulator Dynamics: Kinematic equations describe a robot's motion without considering the forces and torques behind it, while dynamic equations explicitly relate force to motion. Robot kinematics defines how joint motion corresponds to the motion of the robot's rigid bodies. Most robots use electric, hydraulic, or pneumatic actuators to apply forces or torques at the joints. Robot dynamics explains how a manipulator responds to these actuator forces, typically through a set of nonlinear, second-order differential equations based on kinematic and inertial properties. These dynamic equations are essential for robot design, simulation, and control algorithms. The dynamic model aids in motion simulation, structural analysis, and control design. Two primary methods to derive the equations of motion are the Lagrange formulation and the Newton-Euler formulation, with the latter providing a recursive model [23].

2.3.1 Lagrange Formulation: Let us suppose that the symbols $q, \dot{q}, \ddot{q}$, and τ denote the n-dimensiuonal vectors of the joint position, joint velocity, joint acceleration and force variables respectively. Where n is the number of the degree of motion freedom of the robot mechanism. Let $q_i, i = 1,2,3, \ldots \ldots n$ are the number generalised coordinates describing the link position of the manipulator or end effectors. The Lagrangian of the robotic system is defined as a function of the generalised coordinates as

$$L = (T - U)$$

Where $T \; and \; U$ denotes the total kinetic energy and potential energy of the system.
The dynamic equations of motion can be developed with the help of the above Lagrangian equation for each generalised coordinate. The Lagrange's or Euler-Lagrange equations for motion are given as follows:

$$\frac{d}{dt}\frac{\partial L}{\partial q_i} - \frac{\partial L}{\partial q_i} = \tau_i , i = 1,2,3 \ldots \ldots n$$

Where τ_i represents the generalised force associated with the generalised coordinates. The matrix of the above Lagrange's equation is given by: $D(q)\ddot{q} + C(q, \dot{q})q + \dot{g}(q) = \tau$
The n Denquit-Hartenberg joint variables serves as the set of generalised coordinates n-linked rigid robot [24].

2.3.2 Newton-Euler's Formulation: Newton-Euler's formulation of the manipulator dynamics model is based on the balance of all the forces acting on the generic link of the manipulator. The final form of Newton-Euler's formulation can be derived with the help of Newton's equations of motion of the centre of mass and Eulers equation for rotational motion of the link. The Newton-Euler's equation of motion for the manipulators' dynamics is given by

$$\tau_i = I_i\dot{\omega}_i + \omega_i \times (I_i\omega_i)$$

Where τ_i is the torque on the link, I_i is the inertia tensor, ω_i is the angular velocity, and $\dot{\omega}_i$ is the angular acceleration of the i^{th} link [25].

2.4 Control: Control is the fundamental aspect of robotics enabling robots to achieve the desired goals. It involves sending of the signals to the robot's actuators, which controls its movements. The control system considers the robot dynamics, the forces and torques acting on it and its environment. There are two different type of control, open loop control and closed loop control. The open loop control commands actuators without the feedback, whereas the closed loop control adjusts the robot based on sensor feedback. The controls system components include sensors, controllers, actuators etc. The objective of the control includes motion control, force control, hybrid force-motion control and, impedance control etc. [99]

2.4.1 Motion Control: The dynamic model of rigid robot manipulators for motion control in robotics is described by the Lagrange's dynamics. Let the robot manipulator have n-links and let vector q of joints variables is given by $q = (q_1, q_2, q_3, \ldots \ldots q_n)^T$.
The following Lagrange's equation gives the dynamic model of the robot's manipulator

$$H(q)\ddot{q} + C(q, \dot{q})\dot{q} + \tau_g(q) = \tau$$

Where $H(q)$ is the inertia matrix, $C(q,\dot{q})$ is the vector of Coriolis and centrifugal forces, $\tau_g(q)$ is the vector of gravity forces and τ is the vector of the joint control inputs.
The dynamical model of the robots' manipulators holds the following relations:

(i) The inertia matrix is a symmetric positive definite matrix expressed as $\lambda_h I_n \leq H(q) \leq \lambda_H I_n$, where λ_h and λ_H are positive constants.
(ii) The matrix $N(q,\dot{q}) = \dot{H}(q) - 2(q,\dot{q})$ is skew symmetric for a particular choice of $C(q,\dot{q})$ and $z^T N(q,\dot{q})z = 0$.
(iii) The matrix $C(q,\dot{q})$ holds $\|C(q,\dot{q})\| \leq C_0\|q\|$, for some bounded constant C_0.
(iv) The gravity torque forces vector holds $\|\tau_g(q)\| \leq g_0$, for some bounded constant g_0.
(v) The mapping $\tau \to \dot{g}$ is passive, which means that there exists $\alpha \geq 0$ such that $\int_0^t \dot{q}^T(\beta)\,\tau(\beta)d\beta \geq -\alpha\,, \forall\, t < \infty$.

2.4.1.1 Joint Space Control/ Operational Space Control: The joint space control and operational space control are two important approaches to control the robot manipulators. In joint space control the control signals are sent directly to the robot's joints and robots' motion is controlled by adjusting the joint angles. The main objective of the joint space control is to design a feedback controller such that the joint coordinate $q(t) \in \mathbb{R}^n$ tracks the desired motion $q_d(t)$ as closely as possible. The dynamics of the robot's manipulator in joint space controls presented by the equation

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + \tau_g(q) = \tau$$

The operational space control is a more indirect approach to controlling a robot's manipulator. In operational space control the control signals are sent to the end effector of robot and the motion of the robot is controlled by adjusting the end effectors position, orientation and velocity. Let us suppose that the Jacobian matrix $J(q) \in \mathbb{R}^{n\times n}$ transforms the joint velocity $\dot{q} \in \mathbb{R}^n$ to the task velocity $\dot{x} \in \mathbb{R}^n$ as follows: $\dot{x} = J(q)\dot{q}$. Then the dynamics of the robot manipulator in operational space control is presented by the following equation

$$A(q)\ddot{x} + B(q,\dot{q})\dot{x} + \mathrm{C}(q) = f_c$$

Where $f_c \in \mathbb{R}^n$ represents the command force in the operational space and $A, B,$ and C are defined as follows:

$$A(q) = J^{-T}(q)\,H(q)J^{-1}(q)$$
$$B(q,\dot{q}) = J^{-T}(q)\,C(q,\dot{q})J^{-1}(q) - A(q)J(q)\,J^{-1}(q)$$
$$\mathrm{C}(q) = J^{-T}(q)\tau_g(q)$$

2.4.1.2 Independent Joint Control: It is a type of joint control in which the control input of every joint only depends on the measurements of the corresponding joint displacement and velocity. In this type of control, the joint is controlled by a single input, single output (SISD) system. The input-output transfer function of this control is given by

$$M(s) = \frac{k_m}{s(1+sT_m)},$$

Wherek_m andT_m are give by $k_m = \frac{G_v}{K_v}$ and$T_m = \frac{R_a J}{K_v K_t}$ are the voltage to velocity gain and time constant.
Based on the output transform function the control action with position and velocity feedback are characterised by the following relations

$$G_p(s) = k_p \text{and} G_v(s) = k_v\left(\frac{1+sT_v}{s}\right)$$

Where $G_p(s)$and$G_v(s)$ correspond to the position and velocity control actions. The symbols $k_p$andk_v are the constant gain matrices.

2.4.1.3 PD and PID Control: There are two widely used feedback control algorithms in robotics namely proportional derivative (PD) and proportion-integral-derivative (PID) control. The aim of these controllers is to minimise the error between the desired actual positions, velocities and forces of the robot manipulator or end effector.

PD control is a simple and effective control algorithm that utilises the proportional and derivative gain terms. It is used for the purpose of tracking trajectories and controlling the robot's arm. PD control uses a linear control scheme based on the system linearization of operating point. The PD control law has the following form,

$$\tau = k_p(q_d - q) - k_v\dot{q} + \tau_g(q)$$

Where k_p and k_v are the positive definite gain matrices and q_d is the desired joint trajectory. When we apply the above control equation to the equation of the robotic manipulator motion, we obtain the closed loop equation of motion with PD control. This equation is given by,

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + k_v\dot{q} - k_p(q_d - q) = 0$$

PID control is an extension of PD control or the addition of an integral gain term to the PD control law. This integral gain term was added to the PD control to deal with the effect of the gravitational forces. The equation for the PID controller has the following form:

$$\tau = k_p(q_d - q) - k_v\dot{q} + \tau_g(q) + k_I\int f(q_d - q)dt$$

Where k_I is the positive definite gain matrix. The equation of motion for the robot's manipulator for PID control is given by,

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + k_v\dot{q} - k_p(q_d - q) + k_I\int f(q_d - q)dt = 0$$

2.4.1.4 Computed Torque and Computed Torque Like Control: Computed torque control is based on the application of the feedback linearization to the non-linear system. The equation for computed torque control or inverse dynamics controller is given by

$$\tau = H(q)v + C(q,\dot{q})\dot{q} + \tau_g(q), \text{Where } v = \ddot{q} \text{ is the auxilliary control input}$$

This control consists of an inner non-linear compensation loop and an outer loop with control signal v.

In computed torque control it is necessary to that the parameters of the model are calculated accurately and the control input is computed in real time. To avoid this difficulty a computed torque like control scheme has been proposed to the and computed torque like controllers are obtained by modifying the computed torque like control as follows:

$$\tau = \hat{H}(q)v + \hat{C}(q,\dot{q})\dot{q} + \hat{\tau}(q)$$

Where the symbol ^ denotes the computed or nominal value and theoretical uncertainty of exact feedback linearisation. The computed torque like control transforms a multi-input multi-output system (MIMO) non-linear robotic system into a simple decoupled linear closed loop system with well-established control design.

In robotics, there are several less prominent control techniques. Adaptive controllers adjust time-varying parameters based on closed-loop system signals. Examples include adapted computed torque control, adaptive inertia-related control, and adaptive passivity-based control. For non-linear robotic systems, stabilizing controllers are used, but stability alone does not guarantee the best controller. Optimal controllers, such as quadratic optimal control, are essential for non-linear systems but can be challenging to implement due to the gap between the system model and reality, which affects performance. To overcome this, a robust control algorithm is required to improve effectiveness.

2.4.2 Force Control: Robots must be able to handle physical contact with their surroundings to successfully perform tasks. Pure motion control is insufficient because it does not account for modelling errors and uncertainties, which can lead to increased contact forces and unstable behaviour. Force feedback and force control are necessary for robots to operate safely and effectively in unstructured environments, in the presence of humans, and in open-ended environments. Force control plays a vital role in achieving robust and versatile behaviour in robotic and enhances human-robot interaction [26].

2.4.2.1 Interaction Control: Successful robotic manipulation requires optimal interaction between the manipulator and the environment. The contact force at the manipulator or end-effector indicates the state of this interaction. Interaction control is essential for manipulating objects, as the end-effector follows specific geometric path constraints during interaction, known as constrained motion. Achieving successful interaction requires accurate modelling of both the robot and the environment, although modelling the environment is challenging. Planning errors can result in contact forces that deviate the end-effector from its desired trajectory, potentially leading to actuator saturation or component damage. Compliant behaviour during interaction is crucial to mitigate this issue [27].

2.4.2.2 Active and Passive Interaction Control: Passive interaction control relies on a robot's inherent compliance, such as flexibility in its structure, joints, or end-effector, to adjust its path based on interaction forces. This method is simple, cost-effective, and faster since it doesn't need force/torque sensors or trajectory changes. Active interaction control, on the other hand, uses a control system to achieve compliance by measuring contact forces and feeding them back to modify the end-effector's trajectory. While more adaptable, it is slower, costlier, and complex [28]. For optimal performance, combining active control with passive compliance helps manage reaction forces effectively. Active interaction control has two types: indirect and direct force control. Indirect force control, like impedance and stiffness control, manages motion without a force feedback loop. Direct force control, such as hybrid force/motion control, uses feedback to control force along constrained directions while managing motion along unconstrained ones [29].

2.4.2.3 Indirect Force Control: The effects of motion control strategy in the presence of the contact force and moment play an important role in the interaction between end-effector of a robot manipulator and its environment. The velocity of the end-effector is given by,

$$v_e = J(q)\,\dot{q}$$

Where $v_e = \left(\dot{P_e}^T, w_e^T\right)$, and P_e , w_e are the translational and angular velocity.

The operational space formulation of the dynamic model of a robot manipulator in contact with the environment is as follows:

$$A(q)\dot{v} + B(q,\dot{q})v_e + \eta(q) = h_c - h_e$$

Where $A(q)$ is the inertia matrix, $B(q,\dot{q})$ is the wrench including centrifugal and Coriolis effect, $\eta(q)$ is the wrench of the gravitational effect, h_c is the equivalent end-effectors wrench corresponding to the input joint torques, these symbols are defined as follows:

$A(q) = (JH^{-1}(q)\,J^T)^{-1}$

$B(q,\dot{q}) = J^{-T}\,C(q,\dot{q})J^{-1} - A(q)JJ^{-1}$

$\eta(q) = J^{-T}g(q)$

$h_c = J^{-T}\tau$, h_c is the equivalent end-effector's wrench corresponding to the input joint toques

$h_e = \left(f_e^T, m_e^T\right)^T$, f_e and m_e does the end-effector apply the force and moment to the environment are the component of the wrench.

2.4.2.4 Stiffness Control: The stiffness control involves specifying a desired a position and orientation along with an appropriate relationship between the end-effectors position and orientation deviation from the desired motion and the force applied to the environment. In the operational space formulation, the orientation and the end-effectors position are described by the vector

$h_e = \left(P_e{}^T, \varphi_e{}^T\right)^T, Where\ \varphi_e$ is the set of Euler's angles obtained from the rotation matrix. Let x_d be the desired position and orientation of the end-effectors for corresponding origin positions P_d and the rotation matrix R_d. Now let us define another matrix A as follows,

$$A(\varphi_e) = \begin{pmatrix} I_3 & 0 \\ 0 & T_{3\times 3}(\varphi_e) \end{pmatrix}$$

Where T is the matrix of the mapping $w_e = T(\varphi_e)\dot{\varphi_e}$. The equation of motion for stiffness control law can be written using the above matrix and is given by,

$$A^{-T}(\varphi_e)k_p\Delta x_{de} - k_p v_e + \eta(q) = h_c$$

The above equation is the stiffness control lar for the robot's manipulator. The symbol $\Delta x_{de} = x_d - x_e$, the end effectors error and $\Delta \dot{x_{de}} = -\dot{x_e} = -A^{-T}(\varphi_e)v_e$ is the velocity error. The matrix k_p plays the role of active stiffness and $k_p{}^{-1}$ plays the role of an active compliance.

The stiffness control involves specifying a desired position and orientation along with an appropriate static relationship between the end-effectors position and orientation deviation from the desired motion and the force applied to the environment.

2.4.2.5 Impedance Control: Impedance control focuses on the regulation of the force interaction between the force and its environment. By allowing robot to its environment in a more compliant and adaptive manner. The impedance of a robot is the measurement of its mass, stiffness and damping and by impedance control robots can achieve the desired position-force relation for safe navigation of complex environments. The control law for the impedance control in the presence of interaction with the environment can be written using the inverse dynamics control law. The impedance control law is given by,

$$A(q)\alpha + B(q,\dot{q})\dot{q} + h_e = h_c$$

Where α is the properly designed input. Let us suppose that $\dot{v_e} = \bar{R}_e{}^T\dot{v_e^e} + \dot{\bar{R}_e}{}^T v_e^e$, where $\bar{R}_e$ is a matrix given as follows:

$$\bar{R}_e = \begin{bmatrix} R_e & 0 \\ 0 & R_e \end{bmatrix}$$

From control law we have the relation $\alpha = \dot{v}_e$and if we choose $\alpha = \bar{R}_e{}^T\alpha^e + \dot{\bar{R}_e}{}^T v_e^e$ then we obtain the relation between $\alpha^e = \dot{v_e^e}$. Where the control input α^e has the meaning of an acceleration to the end effectors frame.

Now let us assume that $\alpha^e = k_m{}^{-1}(\dot{v_d^e} + k_D\Delta v_{de}^e + h_d^e - h_e^e)$ then the expression for the closed loop system is given by,

$$k_m\Delta\dot{v}_{de}^e + k_D\Delta v_{de}^e + h_\Delta^e = h_e^e,$$

The above equation describes the dynamic behaviour of the controlled end-effector and interpreted as the generalised impedance. where k_m and k_D are the positive definite matrix, $\Delta\dot{v}_{de}^e = \dot{v}_d^e - \dot{v}_e^e$, and $\Delta v_{de}^e = \Delta v_d^e - \Delta v_e^e$ are the acceleration and velocity differences of the desired frame. The symbols v_d^e , $\dot{v}_d^e$ denotes the velocity and acceleration of the desired frame respectively and h_Δ^e is the elastic wrench [30].

2.4.2.6 Hybrid Force-Motion Control: Hybrid force-motion control (HFC) combines the motion control and force control for handling the contact of robots with the environment. HFC controls the robot position, velocity, and force for precise and stable interaction with environment. The approaches used for HFC are acceleration resolved based, velocity based, and passivity based. The acceleration resolved-based and passivity based approach can be applied for both rigid and compliance environment [31]. The acceleration resolved based approach focuses at the decoupling and the linearization of the non-linear dynamics at the acceleration level using the inverse dynamics control law. The passivity-based approach exploits the passivity properties of the dynamic model of the manipulator that also holds for the constrained dynamic model. The acceleration resolved-based approach and the passivity based approach needs modification for the current robot manipulators. The velocity-based control can be obtained through the impedance control if the contact is sufficiently compliant. The control law for impedance control for the closed loop dynamics of a motion-controlled robot can be approximated to obtain the control law for velocity resolved-based control [32].

Conclusion: The integration of mathematics in biomedical robotics has ushered in a transformative era of healthcare delivery, enhancing treatments, diagnostics, and patient monitoring through the fusion of mathematical principles, robotics, and artificial intelligence. This chapter explored the pivotal role mathematics plays in various aspects of biomedical robotics, from surgical interventions to rehabilitation and diagnostics. Mathematics provides the foundation for accurate spatial representations, dynamic modelling, and control algorithms, enabling medical robots to interact seamlessly with biological systems and deliver personalized care. Sensor fusion and signal processing techniques allow robots to interpret complex data and respond to real-time patient changes, improving decision-making capabilities. In surgical robotics, mathematical precision supports minimally invasive procedures, enhancing accuracy and enabling complex operations with smaller incisions. In rehabilitation, mathematical models enable personalized therapies, assisting patients in motor recovery. Additionally, mathematical algorithms aid in early disease diagnosis and optimize patient care, while integration with telemedicine expands access to healthcare. Emerging techniques in machine learning and AI will continue to enhance medical robots, making them more autonomous and human-centric. Despite these advancements, challenges related to accuracy, safety, ethics, and regulatory constraints remain. Interdisciplinary collaboration and responsible innovation are essential to fully harness the potential of biomedical robotics. The ongoing convergence of these technologies promises improved patient outcomes, greater precision, and personalized interventions, making robotic-assisted therapies and precision medicine integral to future healthcare practice.

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