



# ROBUST OPTIMIZATION OF STORAGE SAVINGS GOING ON NETWORKS USING ROTOR BEARING SYSTEMS THROUGH GENETIC ALGORITHM

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**Abstract:** - We recommend an come up to address data uncertainty for disconnected optimization and network flow problems that allows scheming the degree of conservatism of the explanation, and is computationally tractable both practically and theoretically. In particular, when both the cost coefficients and the data in the constraints of an integer programming problem are subject to uncertainty, we propose a robust integer programming problem of moderately larger size that allows controlling the degree of conservatism of the solution in terms of probabilistic bounds on constraint violation. When only the cost coefficients are subject to uncertainty and the problem is a 0 – 1 discrete optimization problem on n variables, then we solve the robust counterpart by solving at most n+ 1 instances of the original problem. Thus, are polynomially solvable. We also show that the robust counterpart of a 0 – 1 discrete optimization problem, remains  $\alpha$ -approximable. Finally, we propose an algorithm for robust network flows that solves the robust counterpart by solving a polynomial number of nominal minimum cost flow problems in a modified network. The problem solves using rotor-bearing systems with this method to minimize the storage optimization on network.

**Keywords:** - Robust Optimization, Rotor Bearing Systems, robust network.

## 1. Introduction

In recent years, much progress has been made in Robust Optimization. While there are many approaches for Robust Optimization in Linear Programming, there are less results for Discrete Optimization and Network Optimization. The first robust approach was proposed by A. Soyster in [5], where he considers Inexact Linear Programming with convex uncertainty sets. His goal is to construct a solution which is feasible for all input data. The approach of this method can directly be applied to min-cost flow problems with uncertain demand. However, the resulting solutions are very conservative.

A less conservative approach was proposed by A. Ben-Tal and A. Nemirovski in [1], who suggest a model for uncertain linear problems with ellipsoidal uncertainty. However, the resulting robust counterpart problems are nonlinear ones, in the form of conic quadratic problems.

In [4], P. Kouvelis and G. Yu propose a general scenario-based approach for Robust Optimization for discrete optimization problems. A drawback of their approach is the fact that the robust counterpart of many polynomially solvable discrete optimization problems becomes NP-hard. The robust approach of D. Bertsimas and M. Sim, as presented in [3], examines the min-cost flow problem with uncertainty in the cost vector. This

approach conserves the network structure, such that the robust counterpart can be solved by solving several min-cost flow problems.

Though in practice the problem of uncertain demand in network flow problems is omnipresent, there still is not a satisfying solution to this problem. In recent years, there has been considerable literature on robust optimization, which has primarily focused on convex optimization problems whose objective functions and constraints were given explicitly and an earlier paper, we proposed a local search method for solving unconstrained robust optimization problems, whose objective functions are given via numerical simulation and may be non convex; see Bertsimas et al. (2009).

In this paper, we extend our approach to solve constrained robust optimization problems, assuming that cost and constraints as well as their gradients are provided. We also consider how the efficiency of the algorithm can be improved if some constraints are convex. We first consider problems with only implementation errors and then extend our approach to admit cases with implementation and parameter uncertainties.

The rest of the paper is structured as follows: In a brief review on unconstrained robust non convex optimization along with the necessary definitions are provided. In the robust local search, as we proposed in Bertsimas et al. (2009), is generalized to handle constrained optimization problems with implementation errors. We also explore how the efficiency of the algorithm can be improved if some of the constraints are convex. In §4, we further generalize the algorithm to admit problems with implementation and parameter uncertainties. In §5, we discuss an application involving a polynomial cost function to develop intuition. We show that the robust local search can be more efficient when the simplicity of constraints are exploited. In §6 we report on an application in an actual health-care problem in intensity-modulated radiation therapy for cancer treatment. This problem has 85 decision variables and is highly non convex.

## 2. Related Works

Addressing data uncertainty in mathematical programming models has long been recognized as a central problem in optimization. There are two principal methods that have been proposed to address data uncertainty over the years: (a) stochastic programming, and (b) robust optimization. As early as the mid 1950s, Dantzig [9] introduced stochastic programming as an approach to model data uncertainty by assuming scenarios for the data occurring with different probabilities. The two main difficulties with such an approach are: (a) Knowing the exact distribution for the data, and thus enumerating scenarios that capture this distribution is rarely satisfied in practice, and (b) the size of the resulting optimization model increases drastically as a function of the number of scenarios, which poses substantial computational challenges. In recent years a body of literature is developing under the name of robust optimization, in which we optimize against the worst instances that might arise by using a min-max objective. Mulvey et al. [14] present an approach that integrates goal programming formulations with scenario-based description of the problem data. Soyster, in the early 1970s, [17] proposes a linear optimization model to construct a solution that is feasible for all input data such that each uncertain input data can take any value from an interval. This approach, however, tends to find solutions that are over-conservative. Ben-Tal and Nemirovski [3, 4, 5] and El-Ghaoui et al. [11, 12] address the over-conservatism of robust solutions by allowing the uncertainty sets for the data to be ellipsoids, and propose efficient algorithms to solve convex optimization problems under data uncertainty. However, as the resulting robust formulations involve conic quadratic problems (see Ben-Tal and Nemirovski [4]), such methods cannot be directly applied to discrete optimization. Bertsimas and Sim [7] propose a different approach to control the level of conservatism in the solution that has the advantage that leads to a linear optimization model and thus, as we examine in more detail in this paper, can be directly applied to discrete optimization models. We review this work in Section 2. Specifically for discrete optimization problems, Kouvelis and Yu [13] propose a framework for robust discrete optimization, which seeks to find a solution that minimizes the worst case performance under a set of scenarios for the data. Unfortunately, under their approach, the robust counterpart of many polynomially solvable discrete optimization problems becomes NP-hard. A related objective is the minimax-regret approach, which seeks to minimize the worst case loss in objective value that may occur. Again, under the minimax-regret notion of robustness, many of the polynomially solvable discrete optimization problems become NP-hard. Under the minimax-regret robustness approach, Averbakh [2]. Howed that polynomial solvability is preserved for a specific discrete optimization problem (optimization over a uniform matroid) when each cost coefficient can

vary within an interval (interval representation of uncertainty); however, the approach does not seem to generalize to other discrete optimization problems. There have also been research efforts to apply stochastic programming methods to discrete optimization (see for example Schultz et al. [16]), but the computational requirements are even more severe in this case.

Our goal in this paper is to propose an approach to address data uncertainty for discrete optimization and network flow problems that has the following features:

- (a) It allows controlling the degree of conservatism of the solution;
- (b) It is computationally tractable both practically and theoretically.

Specifically, our contributions include:

(a) When both the cost coefficients and the data in the constraints of an integer programming problem are subject to uncertainty, we propose, following the approach in Bertsimas and Sim [7], a robust integer programming problem of moderately larger size that allows to control the degree of conservatism of the solution in terms of probabilistic bounds on constraint violation.

(b) When only the cost coefficients are subject to uncertainty and the problem is a 0 – 1 discrete optimization problem on  $n$  variables, then we solve the robust counterpart by solving  $n+1$  nominal problems. Thus, we show that the robust counterpart of a polynomially solvable 0 – 1 discrete optimization problem remains polynomially solvable. In particular, robust matching, spanning tree, shortest path, matroid intersection, etc. are polynomially solvable. Moreover, we show that the robust counterpart of an NP-hard  $\alpha$ -approximable 0–1 discrete optimization problem remains  $\alpha$ -approximable.

(c) When only the cost coefficients are subject to uncertainty and the problem is a minimum cost flow problem, then we propose a polynomial time algorithm for the robust counterpart by solving a collection of minimum cost flow problems in a modified network.

In Section 2, we present the general framework and formulation of robust discrete optimization problems. In Section 3, we propose an efficient algorithm for solving robust combinatorial optimization problems. In Section 4, we show that the robust counterpart of an NP-hard 0 – 1  $\alpha$ -approximable discrete optimization problem remains  $\alpha$ -approximable. In Section 5, we propose an efficient algorithm for robust network flows. In Section 6, we present some experimental findings relating to the computation speed and the quality of robust solutions. Finally, Section 7 contains some remarks with respect to the practical applicability of the proposed methods.

### 3. Proposed Methodology

In this heuristic function for Optimization, where the extreme of the function (i.e., Minimal or maximal) cannot be established analytically. A population of potential solutions is refined iteratively by employing a strategy inspired by Darwinist evolution or natural selection. Genetic Algorithms promote “survival of the fittest”. This type of heuristic has been applied in many different fields, including construction of neural networks and finance.

We represented the parameters of a trading rule with a one-dimension vector that is called “chromosome”, each element is called a “gene”, and all of the “chromosomes” are called “population”. Here, each gene stands for a parameter value; each chromosome is the set of parameters of one trading rule.

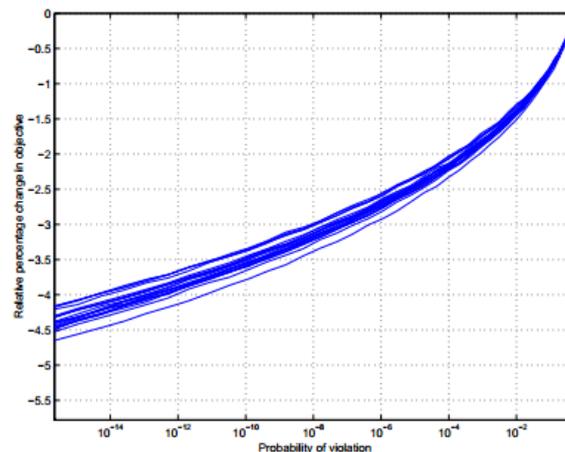
Generally, genetic operations include: “crossover”, “Mutation” and “selection” procedure for process.

1. Initialize Population: Producing a number of individuals randomly, each individual is a chromosome which is a length array,  $n$  is the number of parameters.

2. Test if one of the stopping criteria (running time, fitness, generations, etc) holds. If yes, stop the genetic procedure.
3. Selection: Select the better chromosomes. It means the profit under these parameters is greater.
4. Applying the genetic operators: such as “crossover” and “mutation” to the selected parents to generate an offspring.
5. Recombine the offspring and current population to form a new population with “selection” operator.
6. Repeat steps 2-5. For finding the robust result, we add a filter onto the Algorithm to remove the single peak points. For each Point, we compute the average of the neighbourhood Points, if its value is far from the average, we will discard it. While we finding the fitness one, we also consider its Neighbourhood points.

#### 4. Performance Evaluation

The objective value vector  $c$  is not subject to data uncertainty. An application of this problem is to maximize the total value of goods to be loaded on a cargo that has strict weight restrictions. The weight of the individual item is assumed to be uncertain, independent of other weights and follows a symmetric distribution. In our robust model, we want to maximize the total value of the goods but allowing a maximum of 1% chance of constraint violation.



**Figure 1: robustness and optimality in twenty instances of the 0-1 problem.**

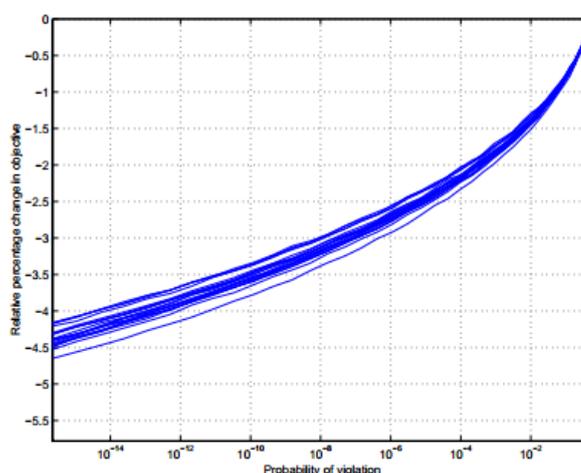
It is interesting to note that the optimal value is marginally affected when we increase the protection level. For instance, to have a probability guarantee of at most 2.57% chance of constraint violation, we only reduce the objective by 2.54%. It appears that in this example we do not heavily penalize the objective function value in order to protect ourselves against constraint violation.

We repeated the experiment twenty times and in Figure 1 we report the trade-off between robustness and optimality for all twenty problems. We observe that by allowing a profit reduction of 9%, we can make the probability of constraint violation smaller than  $10^{-3}$ . Moreover, the conclusion did not seem to depend a lot on the specific instance we generated.

$\Gamma$	Violation Probability	Optimal Value	Reduction
0	1	5592	0%
2.8	$4.49 \times 10^{-1}$	5585	0.13%
36.8	$5.71 \times 10^{-3}$	5506	1.54%
82.0	$5.04 \times 10^{-9}$	5408	3.29%
200	0	5283	5.50%

**Table 1: Robust Solutions.**

We repeated the experiment twenty times and in Figure 2 we report the trade-off between robustness and optimality for all twenty problems. We observe that by allowing a profit reduction of 2%, we can make the probability of constraint violation smaller than  $10^{-3}$ . Moreover, the conclusion did not seem to depend a lot on the specific instance we generated

**Figure 2: The robust sorting problem**

## 5. Conclusion

We experience so as to the projected advance has the possible of being almost useful especially intended for combinatorial optimization and network flow problems that are subject to cost uncertainty. Unlike all other approaches that create robust solutions for combinatorial optimization problems, the proposed approach retains the complexity of the nominal problem or its approximability guarantee and offers the modeller the capability to control the trade-off between cost and robustness by varying a single parameter  $\Gamma$ . For arbitrary discrete optimization problems, the increase in problem size is still moderate, and thus the proposed approach has the potential of being practically useful in this case as well.

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