



# DYNAMIC ANALYSIS OF TWO LINK ROBOT MANIPULATOR FOR CONTROL DESIGN USING COMPUTED TORQUE CONTROL.

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## Abstract

Due to their advantage of high speed, accuracy and repeatability, robot manipulators have become major component of manufacturing industries and even now a days they become part of routine life. Two link robot manipulator is a very basic classical and simple example of robot followed in understanding of basic fundamentals of robotic manipulator. The equation of motion for two link robot is a nonlinear differential equation. As the closed form solutions are not available we have to use numerical solution. Because of these uncertainties and nonlinear behaviour, it's a challenging task to control the motion of robot manipulator at accurate position. Here we focused mainly on control of robot manipulator to get the desired position using computed torque control method. After deriving the equation of motion, control simulation is represented using MATLAB.

**Keywords:** Robot manipulator, Dynamic Control, Computed torque control, Simulation.

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## 1. Introduction

It is well known that robot manipulators are highly nonlinear, dynamically coupled and time-varying systems which are extensively used in industries. The robotic manipulators are generally subjected to uncertainties (Amol et al, 2011). Because of these uncertainties and nonlinear behavior, it's a challenging task to control the motion of robot manipulator at accurate position. For the accurate work any robot has to follow predefined trajectories as close as possible. Which makes trajectory tracking control a most important task in control of robotic manipulator (Korean Ward, 2011).

Looking to the above control requirement of robotic manipulator, researches have been reported on robotic control methods such as proportional-integration-derivative (PID) control (J.Y.S.Luh,1983),PD control(J.Y.S.Luh,1983), PI control, feed-forward compensation control (Pradeep & Kanade,1988), adaptive control (J.J.E. Slotine & W. Li ,1988), variable structure control (Mark W.Spong & M.Vidyasagar,2004,p

247), computed torque control (Pradeep & Kanade,1988). In recent years, much attention has been paid to the use of PID control and computed torque control individually for robotic manipulators.

Two degree of freedom robot manipulator acts like human arms. To understand the complexity of human arms it's very easy to take example of two degree of freedom robot manipulator movement (John M Hollerbach, 1982). The total energy which is sum of kinetic energy and potential energy of the two link system are defined and used to form Lagrangian equations. Finally to define the torque applied on each link these equations are used (Jolly Shah et al, 2012, Stephen Dodds, 2012, Duy Nguyen-Tuong, Jan Peters, 2008).

## 2. Dynamics of Two-Link Manipulators:

Here for the analysis purpose we had used two degree of freedom planer robot manipulator shown in Figure 1.

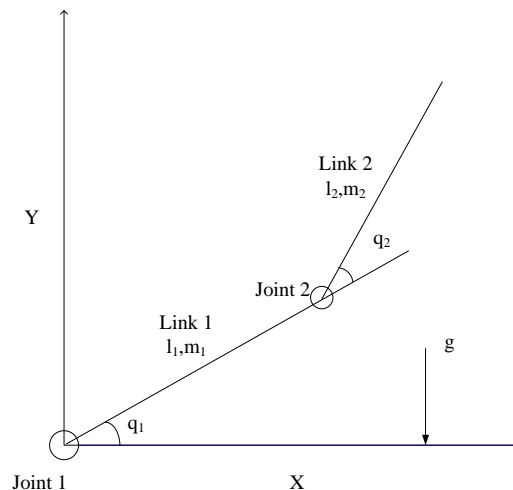


Figure 1 Two Degree of Freedom Robot manipulator

Let us introduce the following notations:

$q_i$  - The joint angle of the joint  $i$ ,

$m_i$  - The mass of link  $i$ ,

$l_i$  - The length of link  $i$ ,

$g$  - The gravitational acceleration.

The forward kinematics equations for 2-DOF robotic manipulator have been derived as [8]:

$$x = l_1 * \cos q_1 + l_2 * \cos(q_1 + q_2) \quad (1)$$

$$y = l_1 * \sin q_1 + l_2 * \sin(q_1 + q_2) \quad (2)$$

In order to simplify we assume that the centre of mass of each link is right at the middle of the link. We will also assume that the gravitational force acts in the negative direction of the Y axis.

Let us choose  $q = (q_1, q_2)^T$  as the generalized coordinates and denote  $T$  the kinetic energy,  $P$  the potential energy and  $L=T-P$  the Lagrangian function.

Using the notations above and taking into account the assumption that we have made, the kinetic energy and potential energy of the system can be expressed as follows:

$$KE = T = \frac{1}{2} (m_1 + m_2) l_1^2 \dot{q}_1^2 + \frac{1}{2} m_2 l_2^2 \dot{q}_2^2 + m_2 l_1 l_2 \dot{q}_1 \dot{q}_2 \cos(q_1 - q_2) \quad (3)$$

$$PE = P = m_1 g l_1 + m_2 g (l_1 \sin q_1 + l_2 \sin q_2) \quad (4)$$

The equations of motion for this manipulator are,

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = \tau_i \quad \text{where } i = 1, 2 \quad (5)$$

Substituting P and T in the formula of L the dynamics equation of the two-link robot manipulator[9]:

$$\tau_1 = m_2 l_2^2 (\ddot{q}_1 + \ddot{q}_2) + m_2 l_1 l_2 c_2 (2\ddot{q}_1 + \ddot{q}_2) + (m_1 + m_2) l_1^2 \ddot{q}_1 - m_2 l_1 l_2 s_2 \dot{q}_2^2 - 2m_2 l_1 l_2 s_2 \dot{q}_1 \dot{q}_2 + m_2 l_2 g s_{12} + (m_1 + m_2) l_1 g s_1 + v_1 \dot{q}_1 \quad (6)$$

$$\tau_2 = m_2 l_2^2 (\ddot{q}_1 + \ddot{q}_2) + m_2 l_1 l_2 c_2 (\ddot{q}_1) + m_2 l_1 l_2 s_2 \ddot{q}_1^2 + m_2 l_2 g s_{12} + v_2 \dot{q}_2 \quad (7)$$

where  $c_1 = \cos(q_1)$ ,  $s_1 = \sin(q_1)$ ,  $s_{12} = \sin(q_1 + q_2)$ ,  $c_2 = \cos(q_2)$ ,  $s_2 = \sin(q_2)$

Which can be rewritten as[10],

$$M(q) \ddot{q} + v(q, \dot{q}) \dot{q} + G(q) + f(q, \dot{q}) \dot{q} = \tau \quad (8)$$

The parameters  $v_i$  and  $k_i$  are viscous and Coulomb friction coefficients, respectively.

The equation can be written in the form of

$$\tau = M \ddot{q} + V \dot{q} + F \dot{q} + G + \tau_d \quad (9)$$

Where,

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

$$M_{12} = M_{21} = m_2 l_1 l_2 \cos(q_2) + m_2 l_2^2$$

$$M_{22} = m_2 l_2^2$$

$$V = \begin{bmatrix} -h \dot{q}_2 & -h \dot{q}_1 - h \dot{q}_2 \\ h \dot{q}_1 & 0 \end{bmatrix}, \text{ with } h = m_2 l_1 l_2 \sin(q_2)$$

$$F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} v_1 \dot{q}_1 \\ v_2 \dot{q}_2 \end{bmatrix}$$

$$G = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} = \begin{bmatrix} m_1 l_1 g \cos(q_1) + m_2 g [l_2 \cos(q_1 + q_2) + l_1 \cos(q_1)] \\ m_2 l_2 g \cos(q_1 + q_2) \end{bmatrix}$$

$$\tau_d = \begin{bmatrix} \tau_{d1} \\ \tau_{d2} \end{bmatrix} = \begin{bmatrix} k_1 \operatorname{sgn}(\dot{q}_1) \\ k_2 \operatorname{sgn}(\dot{q}_2) \end{bmatrix}$$

### 3. Design steps for Computed Torque Control of Robot Manipulators:

Any mechanical system having translation motion is described as shown in Figure 2.

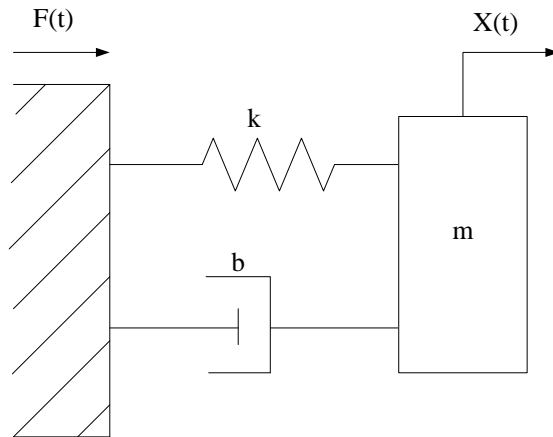


Figure 2 Mechanical systems with translational motion

Open loop dynamic equation for such type of system will be given by following equation,

$$F = mx'' + bx' + kx \tag{10}$$

**Step 1**

Assuming that we don't know value of m, b, k.

In model based control method m, b, k are calculated using real time calculations. Thus system will be simplified to mass only system.

Applying model based control law to equation (10) become,

$$f = mf_s + bx' + kx$$

Where,

$f_s = x''$ , which is new input to system as shown in Figure 3.

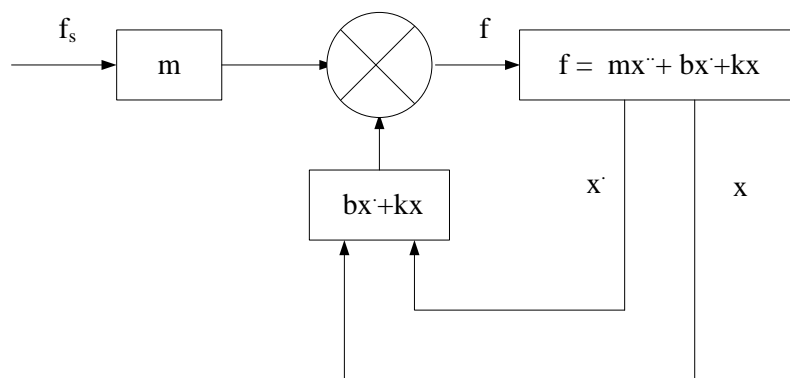


Figure 3 Structure of computer torque control

**Step 2**

If we apply servo control law to above equation, it will get converted into computed torque control system.

Total system block diagram is shown in Figure 4

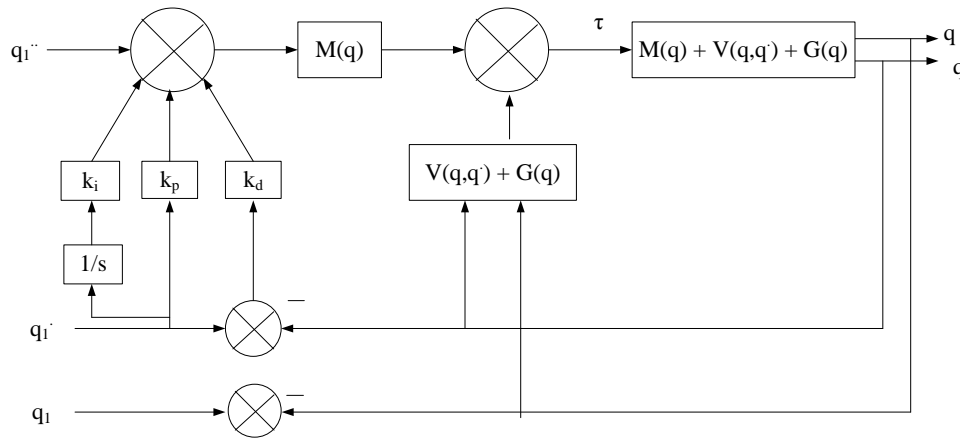


Figure 4 PID feedback loop computed torque controller

Thus total control law is defined as,

$$\tau = M(q)(\ddot{q}_d + k_d \dot{e} + k_p e + k_i \int e) + V(q, \dot{q}) + G(q)$$

Where  $k_d, k_p$  are control gain.

#### 4. Simulation and Results:

For simulation purpose following values are taken in MATLAB Programming.

Total Simulation Time =  $T_1 = 10$  sec and  $T_2 = 120$  sec; Sampling Time =  $T = 0.002$ sec;

mass =  $m = [2; 2]$  unit;

$l_1 = 1$  unit;  $l_2 = 1$  unit;

grav = 9.8 unit;

Gains:

$K_d = [1000 \ 0; 0 \ 1000]$ ;  $K_p = [100 \ 0; 0 \ 100]$ ;

Simulation is done to confirm the results of the proposed control scheme. Simulation is done for first 10sec and then for second time 120sec, and at every 0.002sec the sample reading was taken to plot the graphs.

Figure 5 shows the position errors for joint 1 and joint 2 for 10 sec and Figure 6 shows the velocity errors for joint 1 and joint 2 for 10 sec.

Figure 7 and Figure 8 shows the comparison for derived and predicted position for link 1 and link 2 respectively for 10 sec.

Figure 9 shows the torque required for joint 1 and joint 2 for control purpose for 10 sec.

Figure 10 shows the position errors for joint 1 and joint 2 for 120 sec and Figure 11 shows the velocity errors for joint 1 and joint 2 for 120 sec.

Figure 12 and Figure 13 shows the comparison for derived and predicted position for link 1 and link 2 respectively for 120 sec.

Figure 14 shows the torque required for joint 1 and joint 2 for control purpose for 120 sec.

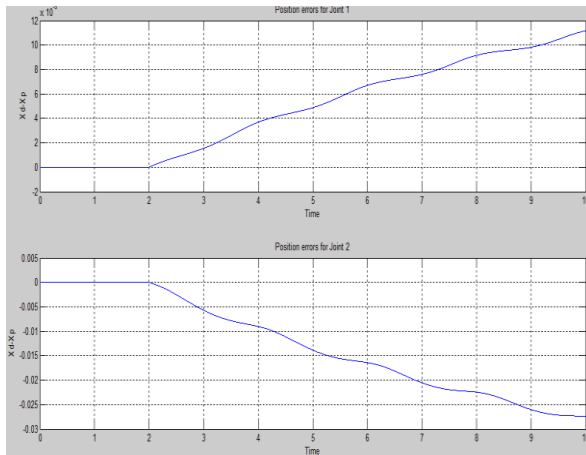


Figure 5 Position errors for joint 1 and joint 2 for 10 sec

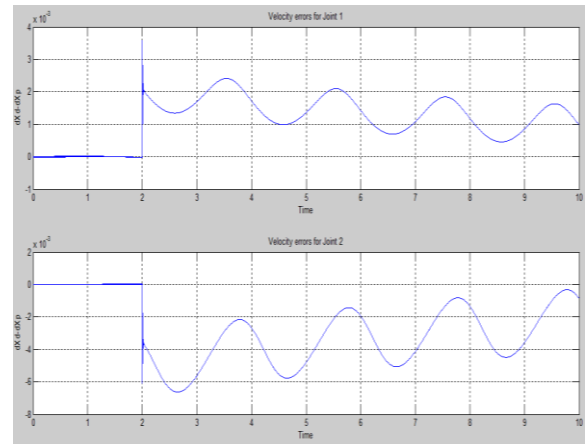


Figure 6 Velocity errors for joint 1 and joint 2 for 10 sec

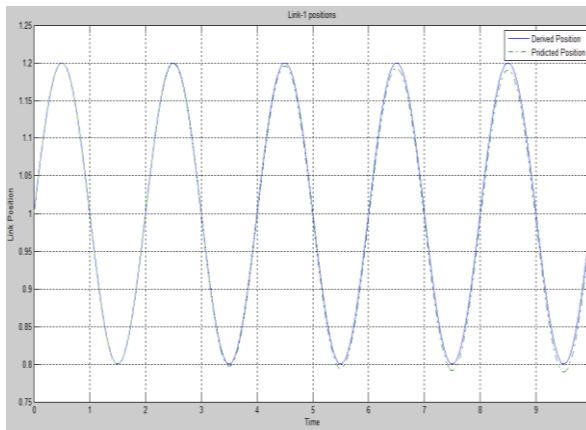


Figure 7 Comparison of derived and predicted position for link 1 for 10 sec.

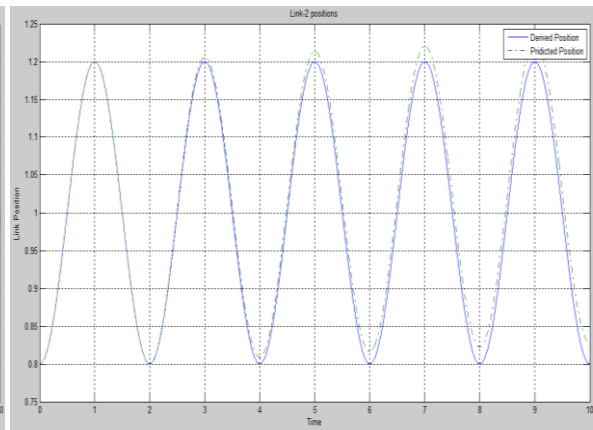


Figure 8 Comparison of derived and predicted position for link 2 for 10 sec.

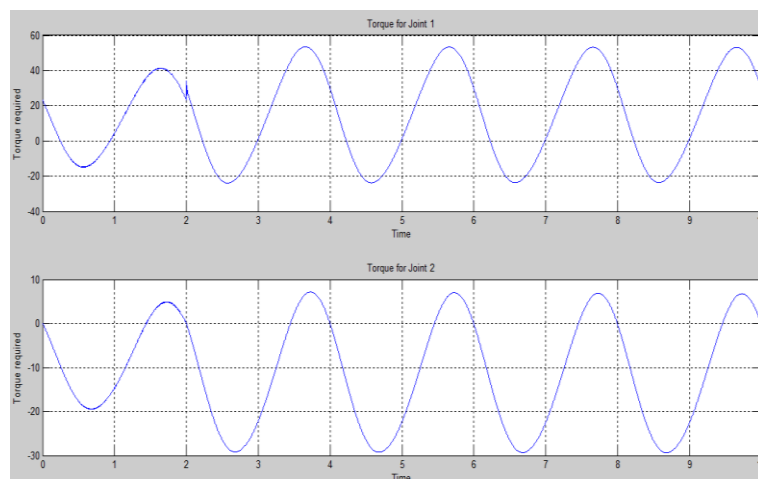


Figure 9 Torque required for joint 1 and joint 2 for 10 sec.

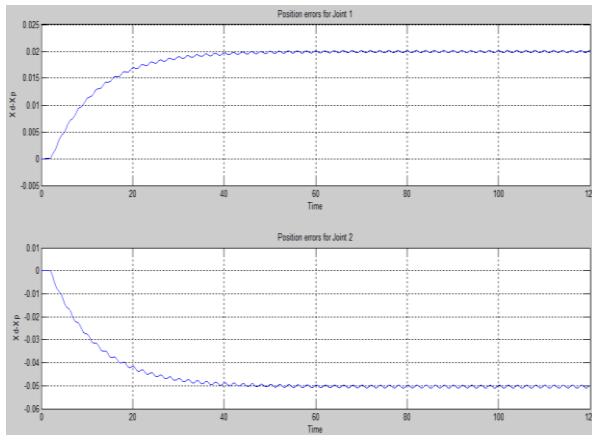


Figure 10 Position errors for joint 1 and joint 2 for 120 sec

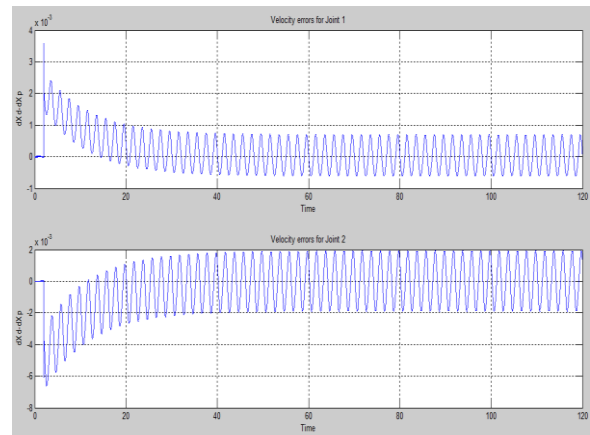


Figure 11 Velocity errors for joint 1 and joint 2 for 120 sec

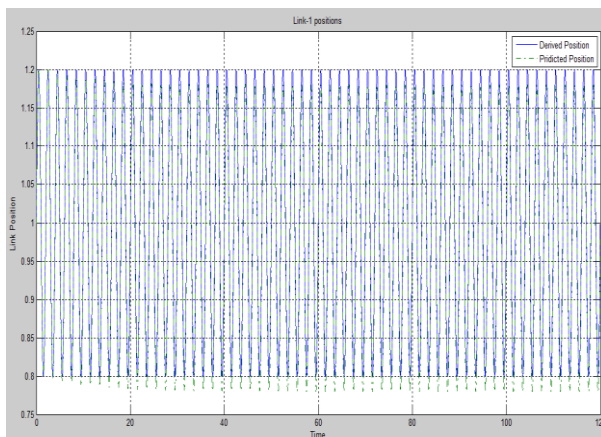


Figure 12 Comparison of derived and predicted position for link 1 for 120 sec

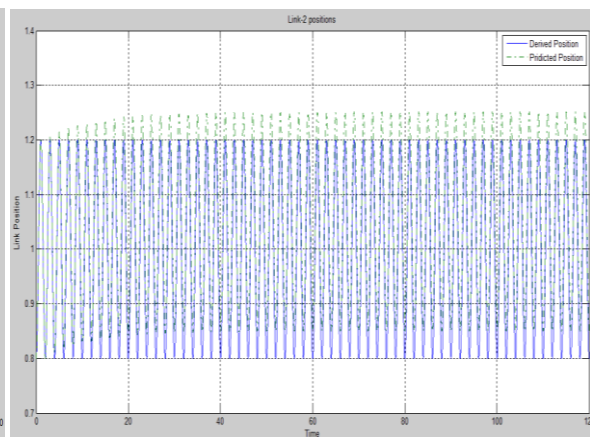


Figure 13 Comparison of derived and predicted position for link 2 for 120 sec.

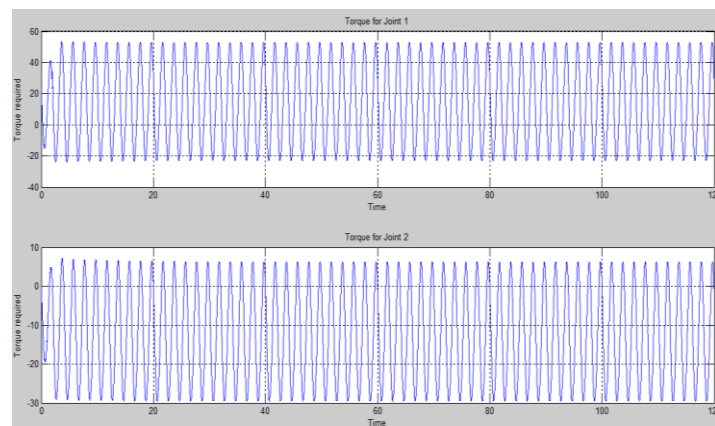


Figure 14 Torque required for joint 1 and joint 2 for 120 sec.

### 5. Conclusion:

In this paper we aimed to apply computed torque controller system for 2 DOF robot manipulator and stimulate the applied controller performance using MATLAB. The simulation results show the validity of the proposed method and give the possibility of a computed torque control for 2 DOF robot manipulator.

Position errors are coming to be less than 1% for almost initial 10 secs but then it's increasing as time going on as shown in simulation graphs. While velocity errors are more for initial 10secs but it's decreasing and getting almost zero as time increasing. Simulations graphs also shows that error between derived and predicted position for link 1 and link 2 is almost around zero for first 10 secs but as time increasing errors are increasing but still it is under control. So if it's required to control the 2 DOF robot for few seconds computed torque controller are suitable but for long time duration it is producing certain errors in performance.

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## Biography

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**Dr. S.S.Rattan** - He is a professor and former head of the Mechanical department at National Institute of Technology, Kurukshetra, India. His areas of interest are Tribology, Fluid Engineering & Machine Design. He received "The Sir Rajendra Nath Mookerjee Memorial Prize" for the best paper in the Journal of Institute of Engineers (Mech.Division) for year 2002. He is also Author of books of Theory of Machines (TMH) & Fluid Mechanics and Machinery (Khanna Publishers). Total no. of Research Publications: 29

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