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A NEW TLM FORMULATION FOR DISPERSIVE MEDIA

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Abstract

In this paper, a new TLM formulation for dispersive media called the piecewise linear current density recursive convolution (PLCDRC) transmission line matrix (TLM) technique is introduced to model the interaction of electromagnetic waves with dispersive media. To confirm the high efficiency and accuracy of this method, the reflection and transmission coefficient magnitudes through a non-magnetized collisional plasma slab are computed and compared to those obtained using the constant recursive convolution (CRC)-TLM method and analytical method.

Keywords: Transmission line matrix algorithm, piecewise linear current density Recursive convolution, dispersive cold plasma media.

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1. Introduction

The transmission line matrix (TLM) method with symmetrical condensed node (SCN) [1-2] remains a robust and efficient numerical tool to model, in the time domain, electromagnetic wave interaction with dispersive media. Over the past two decade, there have been numerous investigations of TLM dispersive media formulations. These include the use of: equations describing the medium behaviour in terms of equivalent node sources [3], the Z transform to describe electric properties of the dispersive media in function of Z variable instead of the angular frequency ω [4], the (CRC) with voltage and current sources [5], the JE convolution (JEC) with voltage sources [6] and recently the

Runge-Kutta Exponential Time Differencing technique (RKETD) approach [7]. In this paper, we propose the PLCDRC-TLM technique to simulate wave propagation in isotropic dispersive "cold plasma". The formulation of this technique is discussed in detail in the next section. The proposed formulation is tested and validated by calculating the reflection and transmission coefficients through a non magnetized air plasma wall in Section 3. Simulation results are compared to those obtained using CRC-TLM method and analytical method.

2-Formulation:

Considering an isotropic cold plasma with collisions. The Maxwell's equations and constitutive relation are given by:

$$\nabla \times \boldsymbol{H} = \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{J} \tag{1}$$

$$\nabla \times \boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{H}}{\partial t} \tag{2}$$

$$\frac{d\boldsymbol{J}}{dt} = \varepsilon_0 \omega_p^2 \boldsymbol{E} - \nu \boldsymbol{J} \tag{3}$$

where, **E** is the electric field, **H** is the magnetic intensity, **J** the polarization current density, ε_0 the permittivity of free space, μ_0 the permeability of free space, ν the electron collision frequency, and ω_p is the plasma frequency. For a time-harmonic dependence, Eq. (3) can be rewritten as:

$$\boldsymbol{J}(\omega) = \varepsilon_0 \frac{\omega_p^2}{j\omega + \nu} \boldsymbol{E}(\omega) = \sigma(\omega) \boldsymbol{E}(\omega) \tag{4}$$

Taking the inverse Fourier transform of (4), we obtain by the use of convolution integral

$$\boldsymbol{J}(t) = \int_0^t \boldsymbol{E}(t-\tau)\sigma(\tau)d\tau \tag{5}$$

$$\sigma(\tau) = \varepsilon_0 \omega_P^2 \exp(-\nu\tau) U(\tau) \tag{6}$$

Where $U(\tau)$ is the unit step function

For $t = n\Delta t$ the polarization current density can be expressed as:

$$J_u(n\Delta t) = J_u^n = \int_0^{n\Delta t} E_u(n\Delta t - \tau)\sigma(\tau)d\tau \tag{7}$$

Where $u = x, y, z$

Assuming that the electric field has a piecewise linear functional dependence over each sub-step.

$$E_u(n\Delta t - \tau) = E_u^{n-m} + \frac{E_u^{n-m-1} - E_u^{n-m}}{\Delta t}(\tau - m\Delta t) \tag{8}$$

Substitution of (8) into (7), after some manipulation

$$J_u^n = \sum_{m=0}^{n-1} [E_u^{n-m} \sigma^m + (E_u^{n-m-1} - E_u^{n-m}) \xi^m] \quad (9)$$

Where

$$\sigma^m = \int_{m\Delta t}^{(m+1)\Delta t} \sigma(\tau) d\tau$$

$$= \frac{\varepsilon_0 \omega_p^2}{\nu} [1 - \exp(-\nu \Delta t)] \exp(-m \nu \Delta t) \quad (10)$$

And

$$\xi^m = \frac{1}{\Delta t} \int_{m\Delta t}^{(m+1)\Delta t} (\tau - m\Delta t) \sigma(\tau) d\tau$$

$$= \frac{\varepsilon_0 \omega_p^2}{\nu^2 \Delta t} [1 - (1 + \nu \Delta t) \exp(-\nu \Delta t)] \exp(-m \nu \Delta t) \quad (11)$$

From Eq. (9), the polarization current density at the (n+1)th time step J_u^{n+1} is

$$J_u^{n+1} = \sum_{m=0}^{n} [E_i^{n+1-m} \sigma^m + (E_u^{n-m} - E_u^{n+1-m}) \xi^m] \quad (12)$$

We then get

$$J_u^{n+1} = J_u^n + (\sigma^0 - \xi^0) E_u^{n+1} + \xi^0 E_u^n - \sum_{m=0}^{n-1} [E_u^{n-m} (\sigma^m - \sigma^{m+1}) + (E_u^{n-m-1} - E_u^{n-m})(\xi^m - \xi^{m+1})$$

$$(13)$$

For simplification we put

$$\Delta \sigma^m = \sigma^m - \sigma^{m+1} \quad (14)$$

$$\Delta \xi^m = \xi^m - \xi^{m+1} \quad (15)$$

And

$$\psi_u^n = \sum_{m=0}^{n-1} [E_u^{n-m} \Delta \sigma^m + (E_u^{n-m-1} - E_u^{n-m}) \Delta \xi^m] \quad (16)$$

Eq.13 can be rewritten as

$$J_u^{n+1} = J_u^n + (\sigma^0 - \xi^0) E_u^{n+1} + \xi^0 E_u^n - \psi_u^n \quad (17)$$

Using Eq. (10) and (11) we get:

$$\Delta\sigma^{m} = \exp(-\nu\Delta t)\Delta\sigma^{m-1} \quad (18)$$

$$\Delta\xi^{m} = \exp(-\nu\Delta t)\Delta\xi^{m-1} \quad (19)$$

So the update equation for ψ_i can be expressed as fellow

$$\psi_u^n = (\Delta\sigma^0 - \Delta\xi^0)E_u^n + \Delta\xi^0 E_u^{n-1} + \exp(-\nu\Delta t)\psi_u^{n-1} \quad (20)$$

Equation (1) can be rewritten as:

$$(\nabla\times\boldsymbol{H})_u^{n+1/2} = \varepsilon_0\frac{E_u^{n+1} - E_u^n}{\Delta t} + \frac{J_u^{n+1} + J_u^n}{2} \quad (21)$$

Then the update equation for E_u and J_u becomes

$$E_u^{n+1} = \frac{1}{\frac{2\varepsilon_0}{\Delta t} + \sigma^0 - \xi^0}[(\frac{2\varepsilon_0}{\Delta t} - \xi^0)E_u^n + 2(\nabla\times\boldsymbol{H})_u^{n+1/2} - 2J_u^n + \psi_u^n] \quad (22)$$

$$J_u^{n+1} = \frac{1}{\frac{2\varepsilon_0}{\Delta t} + \sigma^0 - \xi^0}[(\frac{2\varepsilon_0}{\Delta t} - \sigma^0 + \xi^0)J_u^n + 2(\sigma^0 - \xi^0)(\nabla\times\boldsymbol{H})_u^{n+1/2} + \frac{2\varepsilon_0}{\Delta t}\sigma^0 E_u^n - \frac{2\varepsilon_0}{\Delta t}\psi_u^n]$$

(23)

The equivalence between the electromagnetic fields $(\mathbf{E},\mathbf{H})$ and the electric quantities (V, I) is governed by

$$E = \frac{V}{\Delta l} \quad (24)$$

$$H = \frac{I}{\Delta l} \quad (25)$$

Where $\mathbf{V}$ and $\mathbf{I}$ are the electric voltage and current, and $\Delta\mathbf{l}$ the TLM node step ,

Equations (18), (20) and (21) can then be rewritten as:

$$\psi_u^n = \frac{1}{\Delta l}[(\Delta\sigma^0 - \Delta\xi^0)V_u^n + \Delta\xi^0 V_u^{n-1}] + \exp(-\nu\Delta t)\psi_u^{n-1} \quad (26)$$

$$V_u^{n+1} = \frac{\Delta l}{\frac{2\varepsilon_0}{\Delta t} + \sigma^0 - \xi^0}[\frac{1}{\Delta l}(\frac{2\varepsilon_0}{\Delta t} - \xi^0)V_u^n + 2(\nabla\times\boldsymbol{H})_u^{n+1/2} - 2J_u^n + \psi_u^n] \quad (27)$$

$$J_u^{n+1} = \frac{1}{\frac{2\varepsilon_0}{\Delta t} + \sigma^0 - \xi^0}[(\frac{2\varepsilon_0}{\Delta t} - \sigma^0 + \xi^0)J_u^n + 2(\sigma^0 - \xi^0)(\nabla\times\boldsymbol{H})_u^{n+1/2} + \frac{2\varepsilon_0}{\Delta t\Delta l}\sigma^0 V_u^n + \frac{2\varepsilon_0}{\Delta t}\psi_u^n]$$

(28)

The use of this set of equations in addition to the application of conservation and continuity laws [8], allows us to obtain the TLM scattering matrix. Thus the SCN with additional ports, give the following expression of the total field at $(n+1)\Delta t$ time step:

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}^{n+1} = \frac{1}{2}\begin{pmatrix} V_1^i + V_2^i + V_9^i + V_{12}^i \\ V_3^i + V_4^i + V_8^i + V_{11}^i \\ V_5^i + V_6^i + V_7^i + V_{10}^i \end{pmatrix}^{n+1} + \frac{1}{2}\begin{pmatrix} 0.5V_{sx} \\ 0.5V_{sy} \\ 0.5V_{sz} \end{pmatrix}^{n+1} \qquad (29)$$

$$\begin{pmatrix} I_x \\ I_y \\ I_z \end{pmatrix}^{n+1} = \frac{1}{2}\begin{pmatrix} V_4^i - V_5^i + V_7^i - V_8^i \\ V_6^i - V_2^i + V_9^i - V_{10}^i \\ V_1^i - V_3^i + V_{11}^i - V_{12}^i \end{pmatrix}^{n+1} \qquad (30)$$

V_{sx}, V_{sy} and V_{sz}, injected into ports 13, 14 and 15, are added to model the dispersive behavior of the non-magnetized plasma and considered as voltage sources. They are calculated according to [5] from the following expression:

$$V_{su}^{n+1} = -V_{su}^{n} - 4\frac{\Delta l \Delta t}{\varepsilon_0} J_u^{n+1} \qquad (31)$$

The implementation of the PLCDRC-TLM algorithm is based on the recursive calculation of the current density J given by the Eq. (28), the update of the voltage sources is calculated from Eq. (31), and the total field in each node is given by the equation (29).

3. Numerical results:

In order to demonstrate the accuracy of the PLCDRC-TLM method, the reflection and transmission coefficients of electromagnetic plane wave with normal incidence through a non magnetized cold plasma slab with a thickness of 1.5 cm are calculated. The computational one dimensional domain is subdivided into 350 cells each of $75\ \mu m$ thick. The plasma slab occupies cells from 51 to 251, free space from 1 to 50 and 252 to 350. The plasma parameters are:

$$\omega_p = 2\pi \times 28.7 \times 10^9\ rad\ /\ s$$
$$\nu = 20 \times 10^9\ rad\ /\ s$$

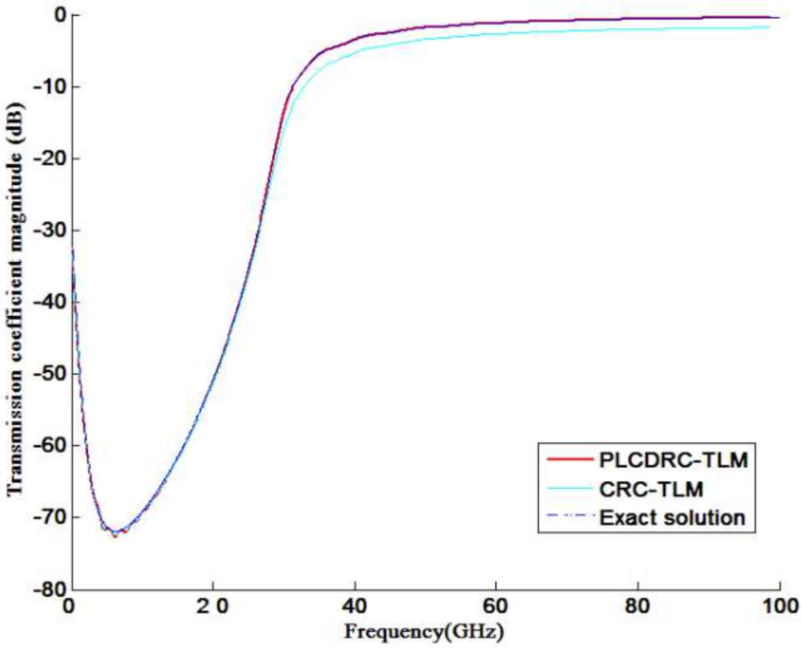

Figure 1: Transmission coefficient magnitude versus frequency

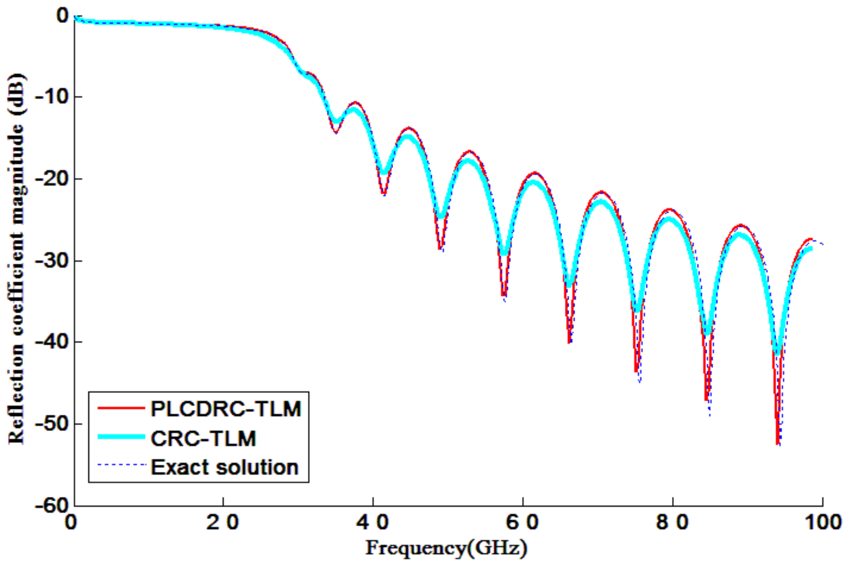

Figure 2: Reflection coefficient magnitude versus frequency

The simulations are allowed to run 20 000 time steps. The transmission and reflection coefficients are calculated trough the Fast Fourier Transform (FFT) of the saved incident and reflected pulses in the time domain.

The magnitudes of transmission coefficients computed using both the PLCDRC method and CRC method and compared to the analytical solution in Fig.1. From Fig.1 we can see that the CRC solution strays significantly from analytical curve above 30 GHz. The PLCDRC solution, however, follows the analytical curve closely at all frequencies.

Fig.2 shows the magnitudes of the reflection coefficients computed using the PLCDRC method, CRC method and analytical solution. We can also see, from fig.2, that the CRC solution strays significantly from the analytical solution while the PLCDRC solution follows the analytical curve closely at all frequencies.

So we can conclude that the PLCDRC-TLM results are in excellent agreement with analytical solutions, which prove the high accuracy of the proposed method.

4. CONCLUSION

In this paper, a new TLM method for modeling dispersive media is derived using the piecewise linear current density recursive convolution. To illustrate the efficiency and accuracy of this method, the transmission and reflection coefficients through an isotropic dispersive non magnetized plasma layer are calculated and compared to the CRC and analytical ones. Results demonstrated excellent agreement between the proposed and analytical model.

REFERENCES

[1] Johns, P.B, “A Symmetrical Condensed Node for the TLM Method” Microwave Theory and Techniques, IEEE Trans. Microwave Theory Tech. Microwave Theory and teIssue Date: Apr 1987.

[2] J. R. Hoefer, “The transmission line matrix method, theory and applications”, IEEE Transactions on Microwave Theory and Techniques, vol. 33, no 10, pp 882-893 1985.

[3] L.R.A.X de Menezes and W. J. R. Hoefer, “Modeling of General Constitutive Relationships in SCN TLM”, IEEE Transactions on Microwave Theory and Techniques, vol. 44, no 6, pp 854-861 1996.

[4] J. Paul, C. Christopoulos, and D. W. P. Thomas, “Generalized material models in TLM-Part 1: Materials with frequency-dependent properties,” IEEE Trans. Antennas, Propagat., vol. 47 , Oct. 1999, pp. 1528–1534.

[5] M. Iben Yaich,., M. .Khalladi, I. Zekik and J. A Morente, “Modeling of frequency-dependent magnetized plasma in hybrid symmetrical condensed TLM method”, IEEE Microw. Wireless Compon. Lett., vol. 12, No. 8, August 2002, pp. 293-295.

[6] R. Abrini, M. Iben Yaich and M. Khalladi, “Efficient modeling of isotropic cold plasma media using JE-TLM method,” IEICE Electron. Express, vol. 4, No. 15, August 10, 2007, pp. 492-497.

[7]S. El Adraoui, M. Bassouh, M. Khalladi, M. Iben Yaich and A. Zugari, “RKETD-TLM modeling of anisotropic magnetized plasma”, Int. Journal of Science and Advanced Technology, Vol. 2, No. 8, pp 81-84, 2012

[8] C. Christopoulos, . The Transmission-Line Modeling (TLM) Method in Electromagnetic. Morgan & Claypool Publishers series, 2006.