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COMPLEX DYNAMICS OF MULTIBROT SETS FOR JUNGCK ISHIKAWA ITERATION

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Abstract

The generation of fractals and study of the dynamics of polynomials is one of the emerging and interesting fields of research nowadays. We introduce in this paper the dynamics of modified multibrot function $z^d - z + c = 0$ for $d \geq 2$ and applied Jungck Ishikawa Iteration to generate new Relative Superior Mandelbrot sets and Relative Superior Julia sets. We have presented here different characteristics of Multibrot function like its trajectories, its complex dynamics and its behaviour towards Julia set are also discussed. In order to solve this function by Jungck –type iterative schemes, we write it in the form of Sz = Tz, where the function T, S are defined as $Tz = z^d + c$ and Sz= z. Only mathematical explanations are derived by applying Jungck Ishikawa Iteration for polynomials in the literature but in this paper we have generated relative Mandelbrot sets and Relative Julia sets.

Keywords: Complex dynamics, Relative Superior Mandelbrot set, Relative Julia set, Jungck Ishikawa Iteration

1. Introduction

In 1918, French mathematician Gaston Julia [21] investigated the iteration process of complex function and attained a Julia set, which is a landmark in the field of fractal theory. The object Mandelbrot set on the other hand was given by Benoit B. Mandelbrot [23] in 1979.

In Mandelbrot's opinion, the turning point in fractal study occurred in 1970-1980 with his research of the Fatou-julia theory of iteration. This theory had last been changed in 1918. Mandelbrot used a computer to investigate a small portion of Fatou-Julia, which he referred to as the μ-map. It was later renamed the Mandelbrot set (M-set) in his honour by Adrian Douady and John Hubbard. The difference between the Mandelbrot set for the family and "its" Julia set is that the structure of the Mandelbrot set varies from locality to locality, while a Julia set is self-similar: the different localities are transformations of each other.
The theory of the Julia sets starts with this question: what can happen when we iterate a point z, that is, form the sequence z_k (k = 0, 1, 2…) where $z_k+1 = f(z_k)$ and $z_0 = z$.
The three possibilities are-

Each sequence of iteration falls within one of these three classes:

- The sequence converges towards a finite cycle of points, and all the points within a sufficiently small neighbourhood of z converge towards the same cycle.
- The sequence goes into a finite cycle of (finite) polygon shaped or (infinite) annular shaped revolving movements, and all the points within a sufficiently small neighbourhood of z go into similar but concentrically lying movements.
- The sequence goes into a finite cycle, but z is isolated having this property, or: for all the points w within a sufficiently small neighbourhood of z, the distance between the iterations of z and w is larger than the distance between z and w.

In the first case the cycle is attracting, in the second it is neutral (in this case there is a finite cycle which is centre for the movements) and in the third case the sequence of iteration is repelling.

The set of point's z, whose sequences of iteration converge to the same attracting cycle or go into the same neutral cycle, is an open set called a Fatou domain of f (z). The complement to the union of these domains (the points satisfying condition 3) is a closed set called the Julia set of f (z).

The Julia set is always non-empty and uncountable, and it is infinitely thin (without interior points). It is left invariant by f (z), and here the sequences of iteration behave chaotically (apart from a countable number of points whose sequence is finite). The Julia set can be a simple curve, but it is usually a fractal.

Fixed point theorem is one of the major tools and it has its diversified applications in the theory of fuzzy mathematics, fractals, theory of games, dynamics programming etc. For a function f having a set X as both domain and range, a fixed point of f is a point x of X for which f(x) =x [20].

2. Multibrot Set

In mathematics, a multibrot set is the set of values in the complex plane whose absolute value remains below some finite value throughout iterations by a member of the general monic univariate polynomial family of recursions .

$$z \mapsto z^d + c.$$

where d ≥ 2. The exponent d may be further generalized to negative and fractional values. So, the collection of point which satisfies the given relation is called Multibrot set. The complex dynamics of Multibrot function, generally known as Multibrot fractal, is a modification of the classic Mandelbrot and Julia sets.

3. Preliminaries

***3.1**: **Ishikawa iteration [2]**:* Let X be a subset of real or complex numbers and T: X→ X for x_0∈ X, we have the sequences $\{x_n\}$ and $\{y_n\}$ in X in the following manner:

$$\left.\begin{aligned} x_{n+1} &= \alpha_n T y_n + (1-\alpha_n) x_n \\ y_n &= \beta_n T x_n + (1-\beta_n) x_n \end{aligned}\right\}$$

where $0 \leq \beta_n \geq 1$ and $0 \leq \alpha_n \geq 1$ and $\alpha_n \& \beta_n$ both convergent to non zero number.

***3.2: Jungck ishikawa iteration [3]**:* Let $(X, \|.\|)$ be a Banach space and Y an arbitrary set. Let S, T: Y→X be two non self mappings such that T(Y) ⊆ S(Y), S(Y) is a complete subspace of X and S is injective. Then for x_o ∈Y, define the sequence $\{Sx_n\}$ iteratively by

$$\left.\begin{aligned} Sx_{n+1} &= \alpha_n T y_n + (1-\alpha_n) Sx_n \\ Sy_n &= \beta_n T x_n + (1-\beta_n) Sx_n \end{aligned}\right\}$$

where n=0, 1 ….. and $0 \leq \beta_n \geq 1$ and $0 \leq \alpha_n \geq 1$ and $\alpha_n \& \beta_n$ both convergent to non zero number.

4. Analysis of superior multibrot set and superior julia set for multibrot

For d=2, the multibrot set converts to Mandelbrot set. We divide our analysis in three categories on the basis of power d to investigate the various characteristics of the superior multibrot set.

Case1. For the positive power or $d \geq 2$

Case2. For the negative power or d<0

Case3. For the non integer power or d, i.e. fractional

Case1. For $d \geq 2$

For d=2, we notice that the superior Multibrot fractal converts to usual superior Mandelbrot fractal. The case is also true for the corresponding superior Julia sets. The Multibrot set for higher values of d are shown in the given figures. The new set has some of the similarities to the superior Mandelbrot set. We analyse that for odd powers the superior Multibrot is symmetrical along both axis and for even powers it is symmetrical only along real axis. We have also analysed that as we increase the value of d, we get d-1 bulbs in each image.

Case2. For d<0

The Multibrot set for negative values of d are given below in figures. We have seen that fixed points can be generated for negative power of d also.The images of superior Julia set for Multibrot set are like a wheel. Inside the wheel we are getting d+1 bulbs in each image.

Case3. For d=fractional

The study for fractional orders of d is given as follows. As we increase the power of d, we see the distortion in the main bulb of Multibrot set. At higher images we are getting amazing images.

5. Relative Superior Mandelbrot sets

5.1 For $d \geq 2$

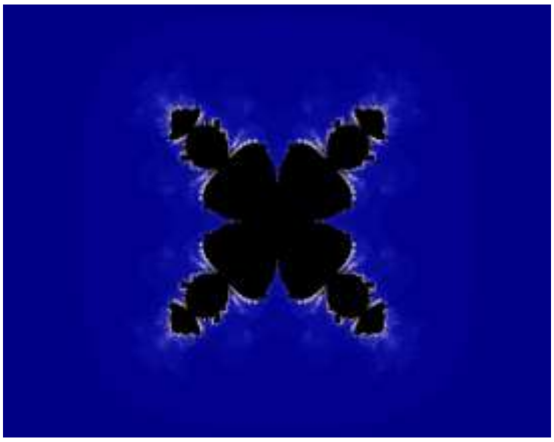

Figure1. α **= 0.5** β **= 0.5 d = 5**

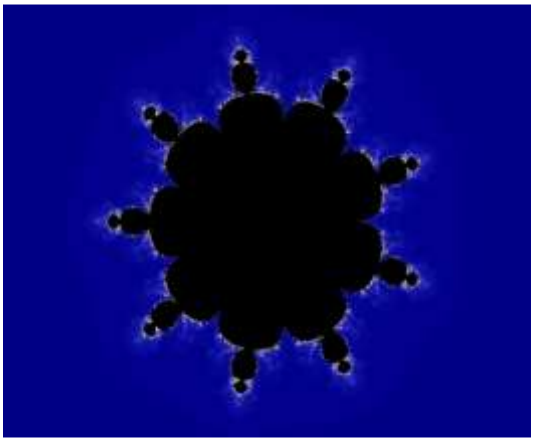

Figure2. α **= 0.8** β **=0.8 d = 10**

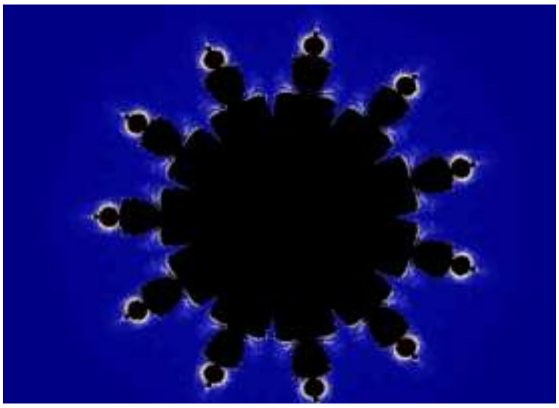

Figure3. α **= 0.5** β **=0.7 d = 12**

5.2 For d<0

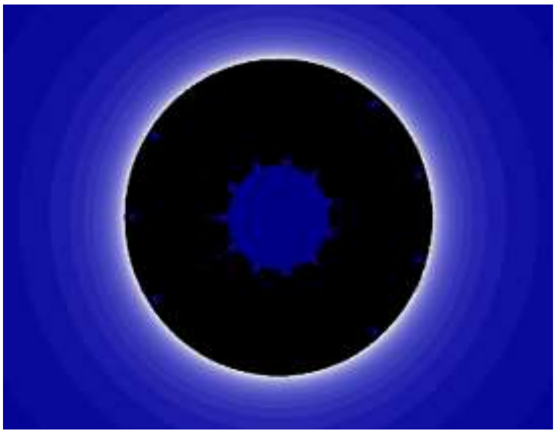

Figure1. α **= 0.2** β **= 0.5 d = -10**

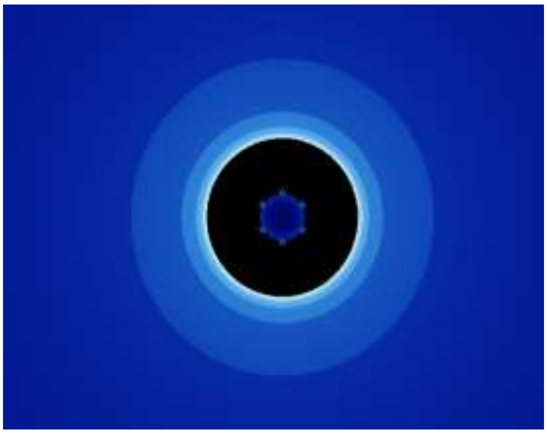

Figure2. α **= 0.5** β **=0.5 d = -5**

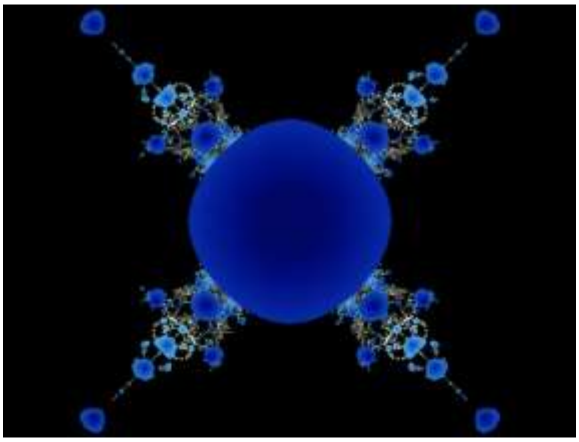

Figure3. α **= 0.5** β **=0.7 d = -3**

5.3 For d=fractional

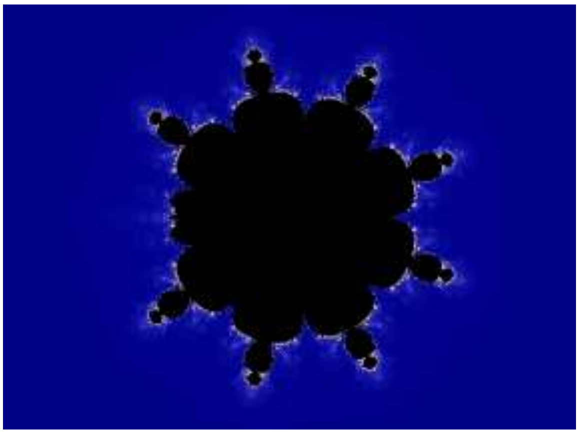

Figure1. α = 0.8 β = 0.8 d = 9.8

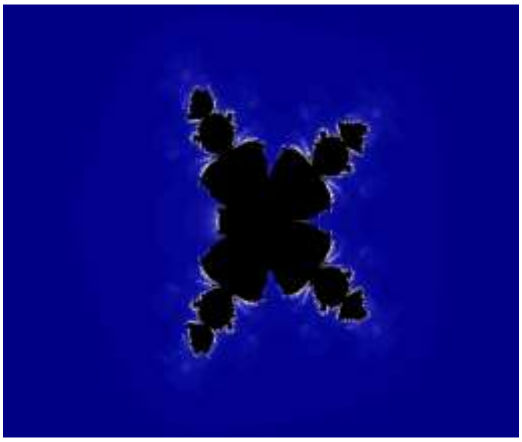

Figure2. α = 0.5 β = 0.5 d = 5.5

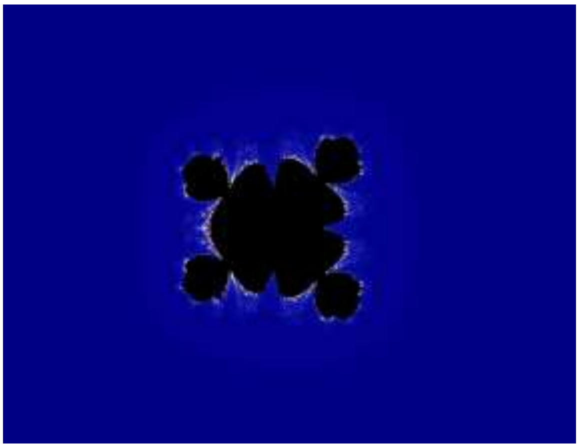

Figure3. α = 0.5 β = 0.8 d = 4.7

6. Relative Superior Julia sets

6.1 For d≥2

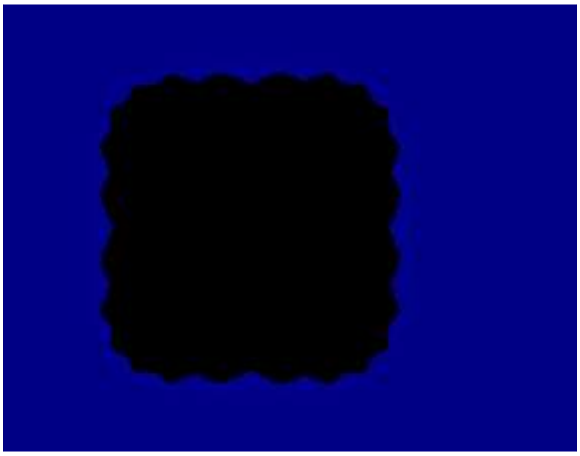

Figure1. α = 0.5 β = 0.5 d = 5
c= -0.03125+0.20625i

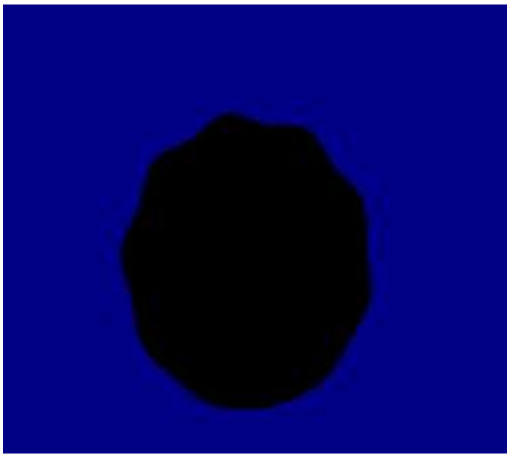

Figure2. α = 0.8 β =0.8 d = 10
c= -0.075+0.10625i

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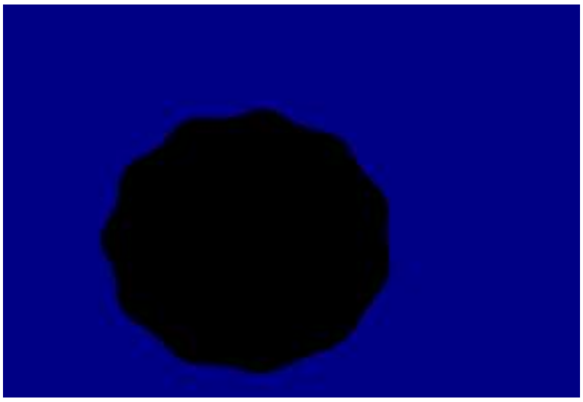


Figure3. $\alpha = 0.5$ $\beta = 0.7$ d = 12
c=-0.0375-0.025i

6.2 For d<0

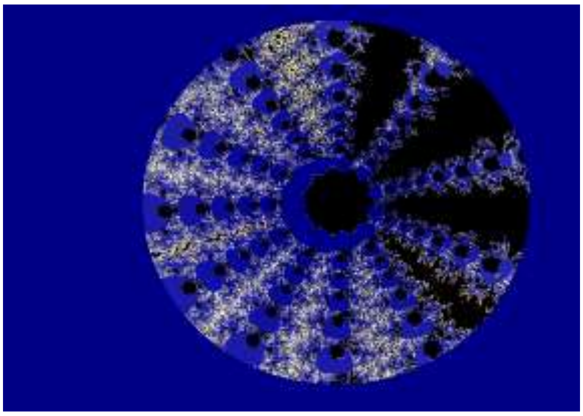


Figure1. $\alpha = 0.2$ $\beta = 0.5$ d = -10
c=0.1875+0.0875i

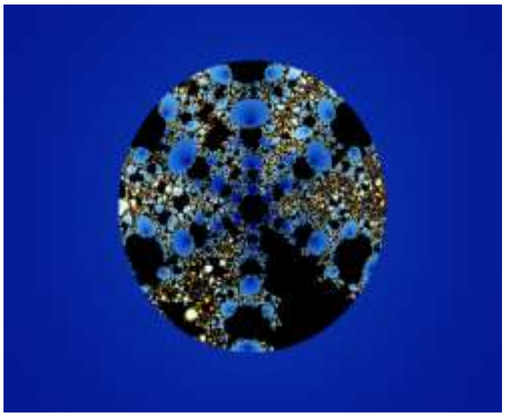


Figure2. $\alpha = 0.5$ $\beta = 0.5$ d = -5
c=0.05+0.18125i

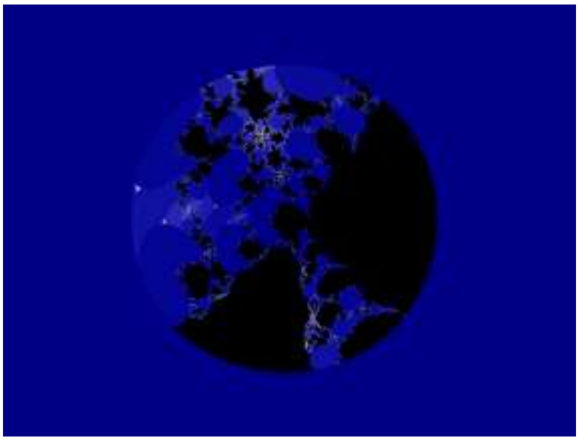


Figure3. $\alpha = 0.5$ $\beta = 0.7$ d = -3
c= 0.325-0.125i

6.3 For d=fractional

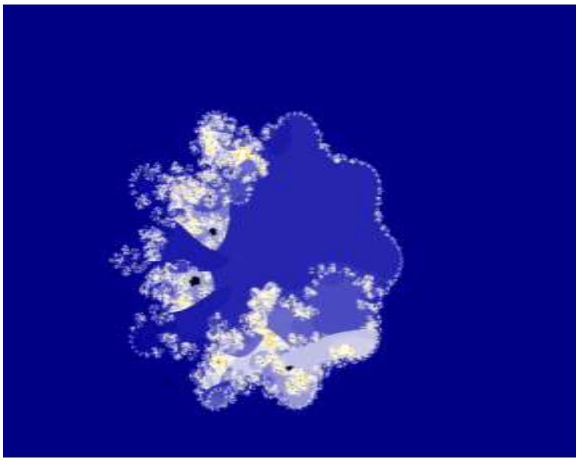


Figure1. α **= 0.8** β **= 0.8 d = 9.8**
c=-0.825+0.01875i

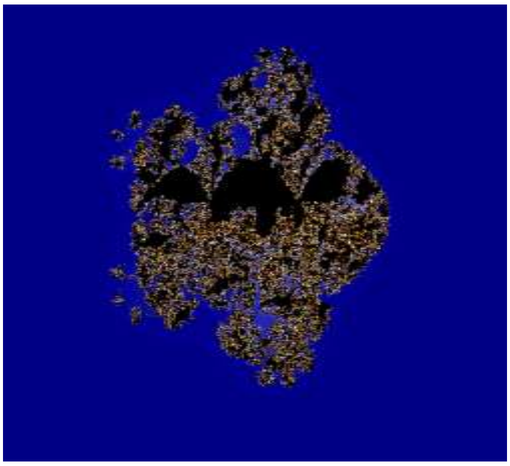


Figure2. α **= 0.5** β **= 0.5 d = 5.5**
c=-0.75-0.7125i

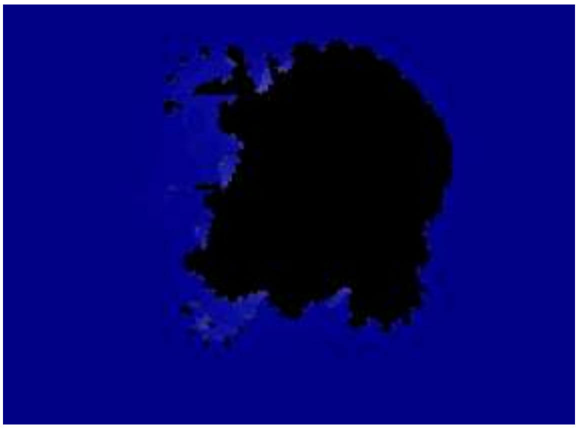
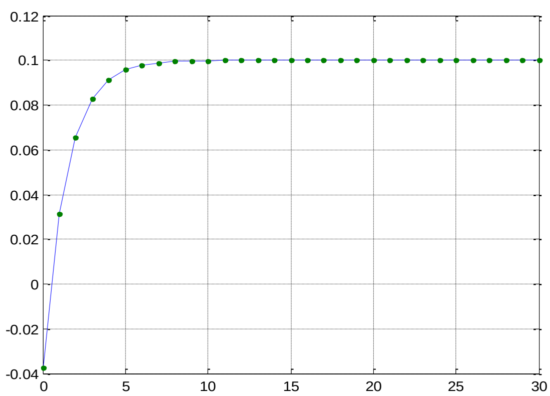


Figure3. α = 0.5 β = 0.8 d = 4.7
c=-0.96875-0.01875i

7. Fixed point

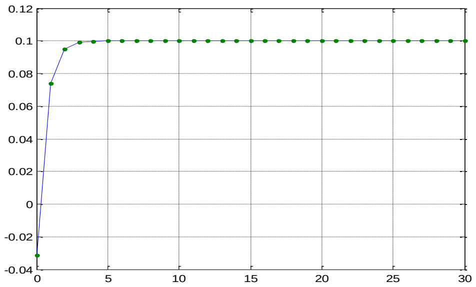
7.1 For d$\geq$2

Table 1: Orbit of F (z) for (z_0=-0.0375-0.025i) at α =0.5, β =0.5, d=5, c=0.1



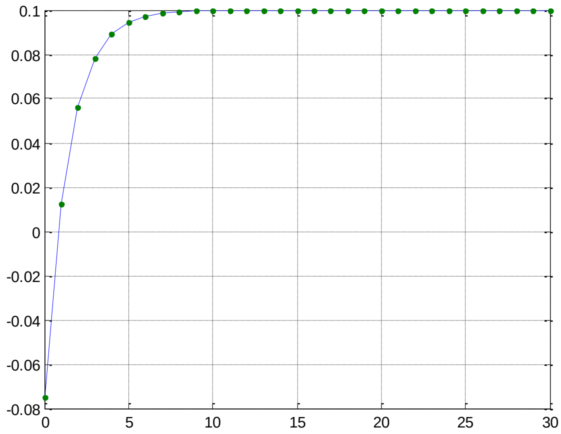
Observation: The value converges to a fixed point after 13 iterations.

Table 2: Orbit of F (z) for (z_0= -0.03125+ 0.20625i) at α =0.8, β =0.8, d=10, c=0.1



Observation: The value converges to a fixed point after 6 iterations.

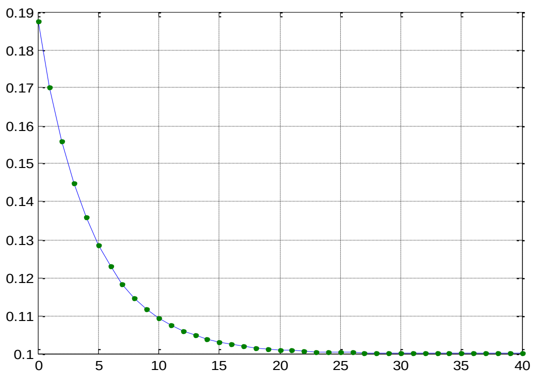
Table 3: Orbit of F (z) for (z_o= -0.075+0.10625i) at α =0.5, β =0.7, d=12, c=0.1



Observation: The value converges to a fixed point after 13 iterations.

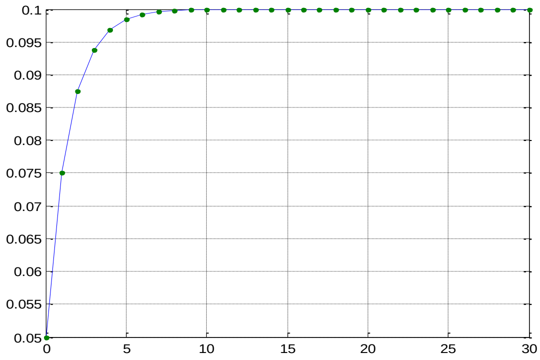
7.2 For d<0

Table 1: Orbit of F (z) for (z_0=0.1875+0.0875i) at α =0.2, β =0.5, d= -10, c=0.1



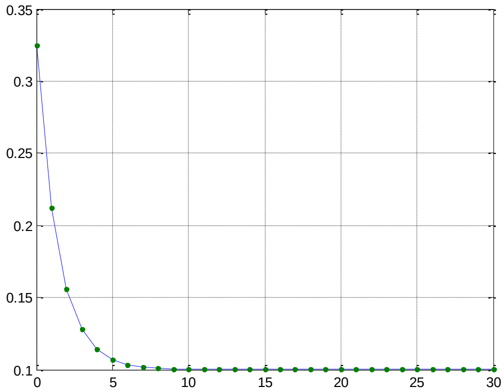
Observation: The value converges to a fixed point after 30 iterations.

Table 2: Orbit of F (z) for (z_0= 0.05+0.18125i) at α =0.5, β =0.5, d= -5, c=0.1



Observation: The value converges to a fixed point after 11 iterations.

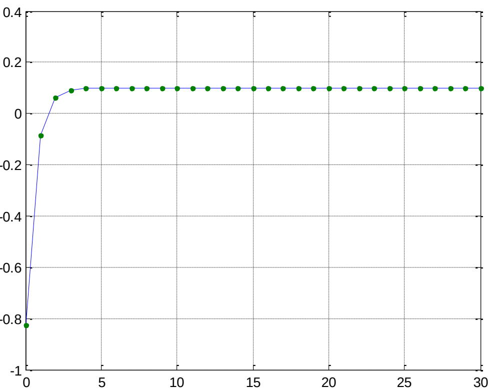
Table 3: Orbit of F (z) for (z_0=0.325-0.125i) at α =0.5, β =0.7, d=-3, c=0.1



Observation: The value converges to a fixed point after 14 iterations.

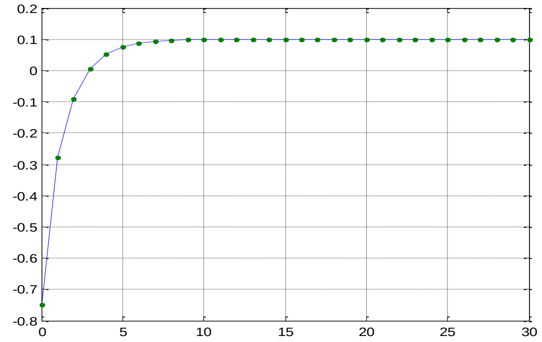
7.3 For d=Fractional

Table 1: Orbit of F (z) for (z_0= -0.825+0.01875i) at α =0.8, β =0.8, d=9.8, c=0.1



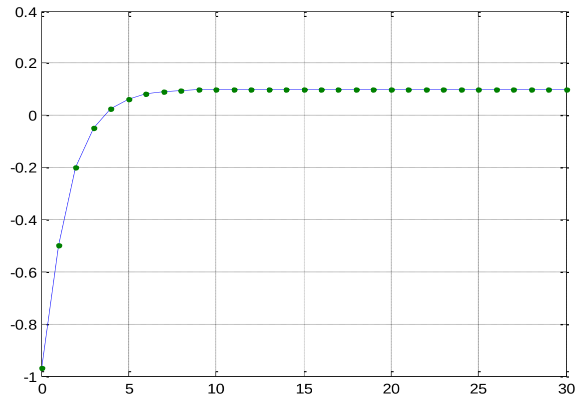
Observation: The value converges to a fixed point after 8 iterations.

Table 2: Orbit of F (z) for (z_0= -0.75-0.7125i) at α =0.5, β =0.5, d=5.5, c=0.1



Observation: The value converges to a fixed point after 15 iterations.

Table 3: Orbit of F (z) for (z_0=-0.96875-0.01875i) at α =0.5, β =0.8, d=4.7, c=0.1



Observation: The value converges to a fixed point after 16 iterations.

8. Conclusion

In this paper we have presented the dynamics and fixed point analysis of Mandelbrot set by using Jungck Ishikawa Iterates for higher, negative and fractional power. In the above discussion we studied the symmetry of the Multibrot sets along axis and noticed the distortion in the main bulb of Multibrot set. We have also presented some spiral shaped Julia sets of Multibrot fractals.

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