INTERNATIONAL JOURNAL OF RESEARCH IN COMPUTER APPLICATIONS AND ROBOTICS **ISSN 2320-7345**

# OSCILLATORY FLOW OF A CONDUCTING VISCOUS FLUID IN A HORIZONTAL COMPOSITE POROUS MEDIUM CHANNEL

**S.Sreenadh[1], A.Parandhama[2] and E.Sudhakara[3]**

[1, 3] Department of Mathematics, Sri Venkateswara University, Tirupati- 517502, India.
**Email:[1]profsreenadh@gmail.com, [3]sudhakare1983@gmail.com**

[2] Department of Mathematics, Sree Vidyanikethan Engineering College, Tirupati, India.
**Email:[2]aparandhama@gmail.com**

Author Correspondence Email: **profsreenadh@gmail.com**, Mobile: +919490418165

## ABSTRACT

Unsteady flow of a conducting fluid in a horizontal composite porous medium is analyzed. A uniform transverse magnetic field of strength $B_0$ is applied perpendicular to the composite channel. The flow in the channel is divided into two regions, namely porous and non-porous regions. The flow in the porous region is modeled using Darcy-Brinkman equation. The viscous and Darcian dissipation terms are also included in the energy equations governing the flow. The nonlinear governing equations are solved analytically using two-term harmonic and non-harmonic functions. The effects of the porous medium parameter, ratio of viscosities, oscillation amplitude, conductivity ratio, Prandtl number and Eckert number on the velocity and the temperature fields are discussed.

*Key words*: Oscillatory flow, Magnetic field, Porous medium, viscous dissipation.

## 1. INTRODUCTION

Viscous flow through or past porous medium is of fundamental importance in petroleum technology, power metallurgy, industrial filtration, ceramic engineering, ground water hydrology and such other fields. In springs of the geothermal region, water is known to be an electrically conducting fluid. The abundant geofluids in the Earth's crust in the geothermal regions has to be brought up to augment fuel output. Earth's surface can be modeled as a natural permeable bed and hence the flow through porous medium has practical importance.

The study of viscous conducting fluids plays a significant role, owing to its practical interest and abundant applications in astro-physical and geo-physical phenomena. The main impetus to the engineering approach to the electromagnetic fluid interaction studies has come from the concept of the hydrodynamics. The flow and heat transfer of electrically conducting fluids in channels and circular pipes under the effect of a transverse magnetic field occurs in MHD generators, pumps, accelerators, and flow meters and have applications in nuclear reactors, filtration, geothermal systems, and others.

Brinkman [1] proposed a non-Darcy law for the flow through highly permeable bed of spherical particles. Beavers and Joseph [2] postulated a slip condition at the nominal surface of the permeable bed. The slip condition has been studied in detail by many other scientists like Richardson [3] for different models. Rudraiah et al. [4] studied the conducting flow of an incompressible viscous fluid over a permeable bed. Rudraiah et al. [5] studied various problems concerning porous media using non-Darcy law. Using this slip condition Rudraiah and Wilfred [6] and Vajravelu et al. [7] analyzed the natural convection in an inclined layer bounded by porous material.

Malashetty and Leela [8] described the magnetohydrodynamic heat transfer in two fluid flows. Chamkha [9] presented analytical solutions for the flow of two-immiscible fluids in porous and non-porous parallel plates. Later on, MHD two-fluid convective flow and heat transfer in composite porous medium was analyzed by Malashetty et al. [10]. Khan et al. [11] investigated for exact solutions for MHD flow of a generalized oldroyd fluid with modified Darcy's law. Umavathi et al. [12] studied unsteady oscillatory flow and heat transfer in a horizontal composite porous medium channel, and they have investigated effects of porous medium and amplitude on the velocity and the temperature. Vajravelu et al. [13] examined the influence of heat transfer on peristaltic transport of Jeffrey fluid in a vertical porous stratum. Vasudev [14] has examined MHD peristaltic flow of Newtonian fluid through a porous medium in an asymmetric vertical channel with heat transfer.

Motivated by these studies, oscillatory flow of a conducting viscous fluid in a composite porous medium channel is investigated. The velocity field, the temperature distribution and the volume flux are obtained in the porous and non-porous regions. The effects of various physical parameters on the velocity and temperature distributions are discussed.

## 2. MATHEMATICAL FORMULATION

Consider the flow of two electrically conducting, immiscible viscous fluids through an infinitely long composite channel, under the influence of uniform transverse magnetic field (Fig. 3.1). The flow region between the plates is divided into two regions. The flow region between the lower plate $y=-h$ and the interface y = 0 is termed as Region 1 (porous matrix region) where as the flow region between the interface y = 0 and the upper plate $y=h$ is designated as Region 2(clear viscous fluid region). The flow in Region 1 is governed by non-Darcy law and the flow in Region 2 is described by Navier-Stokes equations. The fluid velocities in the regions1 and 2 are $u_1$ and $u_2$. The fluid dynamic viscosities in the Regions 1 and 2 are $\mu_1$ and $\mu_2$ respectively. The magnetic field $B_0$ is applied perpendicular to the plates and the induced magnetic field is assumed to be negligible. The following assumptions are made in the analysis of the problem.

(a) The flow in both regions of the channel is assumed to be driven by a common

Pressure gradient $\left(\frac{\partial p}{\partial x}\right)$ and the temperature gradient $\Delta T = T_{\omega_1} - T_{\omega_2}$.

(b) The flow is unsteady and fully developed.

(c) The lower and upper plates are maintained at constant different temperature $T_{w1}$ and $T_{w2}$ where $\left(T_{\omega_1} < T_{\omega_2}\right)$.

(d) The thermo-physical properties of the fluid and the effective properties of the porous medium are assumed to be constant.

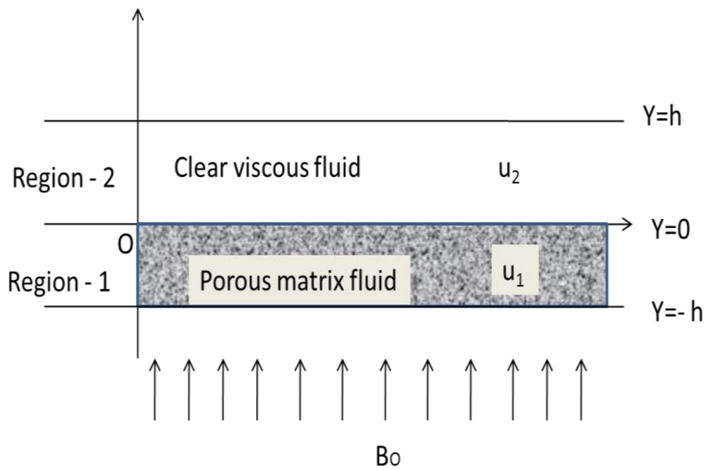


Fig. 1 Physical model.

With the assumptions mentioned above, the equations of motion and the equations of energy are

$$\rho_0\left[\frac{\partial u_i}{\partial t}+v_i\frac{\partial u_i}{\partial y}\right]=\chi_\mu\frac{\partial^2 u_i}{\partial y^2}-\frac{\partial p}{\partial x}-\chi\frac{\mu}{s}u_i-\sigma_e\, B_0^2\,u_i \qquad (i=1,2) \qquad (1)$$

$$\rho_0\, c_p\left[\frac{\partial T_i}{\partial t}+v_i\frac{\partial T_i}{\partial y}\right]=\chi_k\frac{\partial^2 T_i}{\partial y^2}-\chi_\mu\left[\frac{\partial u_i}{\partial x}\right]^2+\chi\frac{\mu}{s}u_i^2 \qquad (i=1,2) \qquad (2)$$

where i=1,2 gives equations for Regions 1 and 2 respectively, u is the x-component of fluid velocity, $v_i$ is the y-component of fluid velocity and T is temperature of the fluid. $\rho_0$, $\mu_i$, $C_p$ and $\sigma_e$ are the fluid density, dynamic viscosity and specific heat at constant pressure and electrical conductivity respectively. The parameter s is the permeability of the porous matrix. The other coefficients appearing in equations (1) and (2) are such that

$$\left.\begin{array}{ll}\chi_\mu=\mu_{eff} & \text{for porous matrix region}\\ \quad =\mu & \text{for clear fluid region}\end{array}\right\},\quad \left.\begin{array}{ll}\chi\ =0 & \text{for porous matrix region}\\ \quad =1 & \text{for clear fluid region}\end{array}\right\},\quad \left.\begin{array}{ll}\chi_k=K_{eff} & \text{for porous matrix region}\\ \quad =K_0 & \text{for clear fluid region}\end{array}\right\}$$

where $K_{eff}$ and $K_0$ are the thermal conductivities in porous and clear fluid regions respectively.

The boundary and interface conditions on velocity for the two fluids can then be written as

$$u_1(-h)=0,\quad u_2(h)=0,\quad u_1(0)=u_2(0),\quad \mu_{eff}\frac{\partial u_1}{\partial y}=\mu\frac{\partial u_2}{\partial y}\quad at\quad y=0 \qquad (3)$$

The thermal boundary and interface conditions are given by

$$T_1(-h)=T_{\omega_1},\quad T_2(h)=T_{\omega_2},\quad T_1(0)=T_2(0),\quad K_{eff}\frac{\partial T_1}{\partial y}=K_0\frac{\partial T_2}{\partial y}\quad at\quad y=0 \qquad (4)$$

The continuity equations of both fluids (1) imply that $v_1$ *and* $v_2$ are independent of y. They can be utmost a function of time alone, we can write [assuming $v_1=v_2=v$ ]

$$v=v_0\left(1+\varepsilon A e^{it\omega}\right) \qquad (5)$$

where A is real positive constant, $\omega$ is frequency parameter and $\varepsilon$ is small such that $\varepsilon A\le 1$. Here it is assumed that the transverse velocity varies periodically with time about a non-zero constant mean $v_0$.

## 3.NON-DIMENSIONALIZATION OF THE FLOW QUANTITIES

It is convenient to introduce the following non-dimensional quantities.

$u_i=\bar{u}_1 u_i^*,\ y=hy^*,\ V=\frac{\upsilon}{h}V^*, V_0=\frac{\upsilon}{h}, t=\frac{h^2}{\upsilon}t^*, \Delta T=T_{w1}-T_{w2},$

$P=\frac{h^2}{\mu_i \bar{U}_i}\left(\frac{\partial p}{\partial x}\right)$, $v=-v_0\left(1+\varepsilon A e^{i\omega t}\right)$ , $\theta=\frac{T_i-T_{\omega_2}}{T_{\omega_1}-T_{\omega_2}}$ , $Ec=\frac{U_0^2}{c_p\,\Delta T}$ (Eckert number)

$\Pr=\frac{\rho_0 \upsilon Cp}{K}$(Prandtl number), $\upsilon=\frac{\mu}{\rho_0}$, and $(v_0=v_1=v_2)$ (6)

In view of the above non-dimensional quantities, the basic equations (1) and (2) and the boundary conditions (3) and (4) can be expressed in non-dimensional form, dropping asterisks, as

$$\frac{\partial u_i}{\partial t}+v\frac{\partial u_i}{\partial y}=A_i\frac{\partial^2 u_i}{\partial y^2}-\left(\chi\sigma^2+M_i^2\right)u_i-P \qquad \text{(i=1, 2)} \qquad (7)$$

$$\frac{\partial \theta_i}{\partial t}+v\frac{\partial \theta_i}{\partial y}=B_i\frac{\partial^2 \theta_i}{\partial y^2}-A_i Ec\left(\frac{\partial u_i}{\partial y}\right)^2+\chi\sigma^2 Ec\left(u_i\right)^2 \qquad \text{(i=1,2)} \qquad (8)$$

where i =1, 2 gives equations for Regions 1 and 2

$A_1=m$, $A_2=1$, $B_1=\frac{K}{\Pr}$, $B_2=\frac{1}{\Pr}$, $\Pr=\frac{\rho_0\upsilon Cp}{K_0}$, $m=\frac{\mu_{eff}}{\mu}$ (ratio of viscosity),

$M_i^2=\frac{\sigma_e B_0 h^2}{\mu_i}$, $\sigma^2=\frac{h^2}{\upsilon}$ and $K=\frac{K_{eff}}{K_0}$ ( ratio of thermal conductivity).

**Region 1**

$$\frac{\partial u_1}{\partial t}+v\frac{\partial u_1}{\partial y}=m\frac{\partial^2 u_1}{\partial y^2}-\left(\sigma^2+M_1^2\right)u_1-P \qquad (9)$$

$$\frac{\partial \theta_1}{\partial t}+v\frac{\partial \theta_1}{\partial y}=B_1\frac{\partial^2 \theta_1}{\partial y^2}-A_1\, Ec\left(\frac{\partial u_1}{\partial y}\right)^2+\sigma^2\, Ec\left(u_1\right)^2 \qquad (10)$$

**Region 2**

$$\frac{\partial u_2}{\partial t}+v\frac{\partial u_2}{\partial y}=\frac{\partial^2 u_2}{\partial y^2}-M_2^2\, u_2-P \qquad (11)$$

.
$$\frac{\partial \theta_2}{\partial t}+v\frac{\partial \theta_2}{\partial y}=B_2\frac{\partial^2 \theta_2}{\partial y^2}-A_2 Ec\left(\frac{\partial u_2}{\partial y}\right)^2 \qquad (12)$$

The non-dimensional form of the hydrodynamic and thermal boundary and interface conditions reduce to

$$u_1(-1)=0,\quad u_2(1)=0,\quad u_1(0)=u_2(0),\quad \mu_{eff}\frac{\partial u_1}{\partial y}=\mu\frac{\partial u_2}{\partial y}\ \ at\ \ y=0 \qquad (13)$$

$$T_1(-1)=T_{\omega_1},\quad T_2(1)=T_{\omega_2},\quad T_1(0)=T_2(0),\quad K_{eff}\frac{\partial T_1}{\partial y}=K_0\frac{\partial T_2}{\partial y}\ \ at\ \ y=0 \qquad (14)$$

## 4. SOLUTION OF THE PROBLEM

The governing momentum and energy equations (9) to (14) are coupled partial differential equations that cannot be solved in closed form. However, they can be reduced to set of ordinary differential equations that can be solved analytically. This can be done by representing the velocity and temperature as

$$u_i(y,t) = u_{i0}(y) + \varepsilon e^{i\omega t} u_{i1}(y) + 0(\varepsilon^2) + \text{------} \qquad i = 1, 2 \qquad (15)$$

$$\theta_i(y,t) = \theta_{i0}(y) + \varepsilon e^{i\omega t} \theta_{i1}(y) + 0(\varepsilon^2) + \text{------} \qquad i = 1, 2 \qquad (16)$$

This is a valid assumption because of the choice of v as defined in equation (5) that the amplitude $\varepsilon A \leq 1$ and $\omega$ is the frequency parameter .

By substituting equations (15) and (16) into (9) to (12) and equating the harmonic and non-harmonic terms and neglecting the higher order terms of $0(\varepsilon^2)$,one obtains the following pairs of equation for $(u_{i0}, \theta_{i0})$ and $(u_{i1}, \theta_{i1})$ where i= 1,2.

**Region 1**

$$m\frac{d^2u_{10}}{dy^2} + \frac{du_{10}}{dy} - (\sigma^2 + M_1^2)u_{10} = P \qquad (17)$$

$$B_1\frac{d^2\theta_{10}}{dy^2} + \frac{d\theta_{10}}{dy} = A_1 Ec\left(\frac{du_{10}}{dy}\right)^2 - \sigma^2 Ec(u_{10})^2 \qquad (18)$$

$$m\frac{d^2u_{11}}{dy^2} + \frac{du_{11}}{dy} - (\sigma^2 + M_1^2 + i\omega)u_{11} = -A\left(\frac{du_{10}}{dy}\right) \qquad (19)$$

$$B_1\frac{d^2\theta_{11}}{dy^2} + \frac{d\theta_{10}}{dy} - i\omega\theta_{11} = A_1\frac{d\theta_{11}}{dy} + 2A_1 Ec\frac{du_{10}}{dy}\frac{du_{11}}{dy} - 2\sigma^2 Ec\, u_{10}\, u_{11} \qquad (20)$$

**Region 2**

$$\frac{d^2u_{20}}{dy^2} + \frac{du_{20}}{dy} - M_2^2 u_{20} = P \qquad (21)$$

$$B_2\frac{d^2u_{20}}{dy^2} + \frac{d\theta_{20}}{dy} = A_2 Ec\left(\frac{du_{20}}{dy^2}\right) \qquad (22)$$

$$\frac{d^2u_{21}}{dy^2} + \frac{du_{21}}{dy} - (M_2^2 + i\omega)u_{21} = -A\left(\frac{du_{20}}{dy}\right) \qquad (23)$$

$$B_2\frac{d^2\theta_{21}}{dy^2} + \frac{d\theta_{21}}{dy} - i\omega\theta_{21} = -A\frac{d\theta_{20}}{dy} + 2A_2 Ec\frac{du_{20}}{dy}\frac{du_{21}}{dy} \qquad (24)$$

Using (15) and (16), the boundary and interface conditions may be written as

$$u_{10}(-1)=0,\quad u_{20}(1)=0,\quad u_{10}(0)=u_{20}(0),\quad \mu_{eff}\frac{\partial u_{10}}{\partial y}=\mu\frac{\partial u_{20}}{\partial y}\quad at\quad y=0 \quad (25)$$

$$u_{11}(-1)=0,\quad u_{21}(1)=0,\quad u_{11}(0)=u_{21}(0),\quad \mu_{eff}\frac{\partial u_{11}}{\partial y}=\mu\frac{\partial u_{21}}{\partial y}\quad at\quad y=0 \quad (26)$$

$$\theta_{10}(-1)=1,\quad \theta_{20}(1)=0,\quad \theta_{10}(0)=\theta_{20}(0),\quad K_{eff}\frac{\partial \theta_{10}}{\partial y}=K_0\frac{\partial \theta_{20}}{\partial y}\quad at\quad y=0 \quad (27)$$

$$\theta_{11}(-1)=1,\quad \theta_{21}(1)=0,\quad \theta_{11}(0)=\theta_{21}(0),\quad K_{eff}\frac{\partial \theta_{11}}{\partial y}=K_0\frac{\partial \theta_{21}}{\partial y}\quad at\quad y=0 \quad (28)$$

The solution of the equations (17) to (24) using the boundary and interface conditions (25) to (28) can be written as:

$$u_{10}=c_1e^{a_1y}+c_2e^{a_2y}-\frac{P}{\left(\sigma_1^2+M_1^2\right)} \quad (29)$$

$$u_{20}=c_3e^{a_3y}+c_4e^{a_4y}-\frac{P}{M_2^2} \quad (30)$$

$$\theta_{10}=c_{11}+c_{12}e^{t_1y}+t_7y+t_8e^{2a_1y}+t_9e^{2a_2y}+t_{10}e^{m_1y}+t_{11}e^{a_1y}+t_{12}e^{a_2y} \quad (31)$$

$$\theta_{20}=c_{13}+c_{14}e^{t_{13}y}+t_{14}e^{a_3y}+t_{15}e^{a_4y} \quad (32)$$

$$\begin{aligned}u_{11}&=e^{e_1y}\left[R_1\cos f_1y+R_5\sin f_1y\right]+e_2e^{a_1y}+e_4e^{a_2y}\\&\quad+i\left\{e^{e_1y}\left[R_2\cos f_1y+R_6\sin f_1y\right]+e_3e^{a_1y}+e_5e^{a_2y}\right\}\end{aligned} \quad (33)$$

$$\begin{aligned}u_{21}&=e^{e_6y}\left[R_3\cos f_2y+R_7\sin f_2y\right]+e_7e^{a_3y}+e_9e^{a_4y}\\&\quad+i\left\{e^{e_6y}\left[R_4\cos f_2y+R_8\sin f_2y\right]+e_8e^{a_3y}+e_{10}e^{a_4y}\right\}\end{aligned} \quad (34)$$

$$\begin{aligned}\theta_{11}&=e^{f_4y}\left[R_{21}\cos f_5y+R_{25}\sin f_5y\right]+e^{a_{21}y}\left[\left(r_{16}+r_{36}\right)\cos f_1y+\left(r_{17}+r_{37}\right)\sin f_1y\right]\\&\quad+e^{a_{23}y}\left[\left(r_{26}+r_{46}\right)\cos f_1y+\left(r_{27}+r_{47}\right)\sin f_1y\right]+e^{e_1y}\left[r_{56}\cos fy+r_{57}\sin f_1y\right]\\&\quad+e^{a_1y}\left[g_5+g_{16}\right]+e^{a_2y}\left[g_6+g_{17}\right]+e^{2a_1y}\left[g_2+g_{14}+g_{11}+g_8\right]\\&\quad+e^{a_2y}\left[g_3+g_9+g_{12}+g_{15}\right]+g_1e^{t_1y}+\left[g_4+g_7+g_{10}+g_{13}\right]e^{a_{22}y}\\&+i\left\{\begin{array}{l}-e^{a_{21}y}\left[\left(r_{18}+r_{38}\right)\cos f_1y+\left(r_{19}+r_{39}\right)\sin f_1y\right]-e^{e_1y}\left[r_{58}\cos f_1y+r_{59}\sin f_1y\right]\\-e^{a_{23}y}\left[\left(r_{18}+r_{38}\right)\cos f_1y+\left(r_{19}+r_{39}\right)\sin f_1y\right]+e^{a_1y}\left[h_5+h_{16}\right]+e^{a_2y}\left[h_6+h_{17}\right]\\+e^{2a_1y}\left[h_2+h_{14}+h_{11}+h_8\right]+e^{2a_2y}\left[h_3+h_{15}+h_{12}+h_9\right]+e^{a_0y}\left[h_4+h_7+h_{10}+h_{13}\right]\\+h_1e^{t_1y}+\dfrac{At_7}{\omega}+e^{f_4y}\left[R_{22}\cos f_5y+R_{26}\sin f_5y\right]\end{array}\right\}\end{aligned} \quad (35)$$

$$\theta_{21} = e^{f_6 y}\left[R_{23}\cos f_7 y + R_{27}\sin f_7 y\right] + e^{f_8 y}\left[l_7\cos f_2 y + l_8 \sin f_2 y\right] + e^{f_9 y}\left[l_{17}\cos f_2 y + l_{18}\sin f_2 y\right]$$
$$+ g_{20}e^{t_{13} y} + g_{21}e^{a_3 y} + g_{22}e^{a_4 y} + g_{23}e^{2a_3 y} + g_{24}e^{2a_4 y} + g_{25}e^{f_{10} y}$$
$$+ i\left\{\begin{array}{l} e^{f_6 y}\left[R_{24}\cos f_7 y + R_{28}\sin f_7 y\right] + e^{f_8 y}\left[l_9\cos f_2 y + l_{10}\sin f_2 y\right] \\ + e^{f_9 y}\left[l_{19}\cos f_2 y + l_{20}\sin f_2 y\right] + h_{20}e^{t_{13} y} + h_{21}e^{a_3 y} + h_{22}e^{a_4 y} + h_{23}e^{2a_3 y} + h_{24}e^{2a_4 y} + h_{25}e^{f_{10} y}\end{array}\right\} \quad (36)$$

The velocities and temperature distributions in the two regions are

$$u_1 = u_{10} + \varepsilon e^{i\omega t}u_{11} \quad (37) \qquad \theta_1 = \theta_{10} + \varepsilon e^{i\omega t}\theta_{11} \quad (39)$$
$$u_2 = u_{20} + \varepsilon e^{i\omega t}u_{21} \quad (38) \qquad \theta_2 = \theta_{20} + \varepsilon e^{i\omega t}\theta_{21} \quad (40)$$

Where

$$a_1 = \frac{-1+\sqrt{1+4m\left(\sigma_1^2+M_1^2\right)}}{2m},\ a_2 = \frac{-1-\sqrt{1+4m\left(\sigma_1^2+M_1^2\right)}}{2m},\qquad a_3 = \frac{-1+\sqrt{1+4M_2^2}}{2},$$
$$a_4 = \frac{-1-\sqrt{1+4M_2^2}}{2},\ r_1 = \sqrt{\left[1+fm\sigma_1^2+4mM_1^2\right]+16m^2\omega^2},\ \theta_1 = Tan^{-1}\theta\left[\frac{4m\omega}{1+4m\left(\sigma_1^2+M_1^2\right)}\right]$$
$$e_1 = \frac{-1+\sqrt{r_1}\cos\left(\frac{\theta_1}{2}\right)}{2m},\ f_1 = \frac{\sqrt{r_1}\sin\left(\frac{\theta_1}{2}\right)}{2m},\ e_2 = \frac{-Ac_1a_1\left[a_1^2+a_1-\sigma_1^2-M_1^2\right]}{\left[a_1^2+a_1-\sigma_1^2-M_1^2\right]^2+\omega^2},$$
$$e_3 = \frac{-Ac_1a_1\omega}{\left[a_1^2+a_1-\sigma_1^2-M_1^2\right]^2+\omega^2},\ e_4 = \frac{-Ac_2a_2\left[a_2^2+a_2-\sigma_1^2-M_1^2\right]}{\left[a_2^2+a_2-\sigma_1^2-M_1^2\right]^2+\omega^2}, r_2 = \sqrt{\left[1+4M_2^2\right]^2+16\omega^2}$$
$$e_5 = \frac{-Ac_2a_2\omega}{\left[a_2^2+a_2-\sigma_1^2-M_1^2\right]^2+\omega^2},\qquad \theta_2 = Tan^{-1}\left(\frac{4\omega}{1+4M_2^2}\right), e_6 = \frac{-1+\sqrt{r_2}\cos\left(\frac{\theta_2}{2}\right)}{2},$$
$$f_2 = \frac{\sqrt{r_2}\sin\left(\frac{\theta_2}{2}\right)}{2}, e_7 = \frac{-Ac_3a_3\left[a_3^2+a_3-M_2^2\right]}{\left[a_3^2+a_3-M_2^2\right]^2+\omega^2}, e_8 = \frac{-Ac_3a_3\omega}{\left[a_3^2+a_3-M_2^2\right]^2+\omega^2}, d_1 = \frac{-P}{\sigma_1^2+M_1^2}$$
$$e_9 = \frac{-Ac_4a_4\left[a_4^2+a_4-M_2^2\right]}{\left[a_4^2+a_4-M_2^2\right]^2+\omega^2},\quad e_{10} = \frac{-Ac_4a_4\omega}{\left[a_4^2+a_4-M_2^2\right]^2+\omega^2},\ d_2 = \frac{-P}{M_2^2},\ d_3 = -de^{a_1},$$
$$d_4 = -e^{a_1-a_2},\ d_5 = -e^{a_4-a_3},\ d_6 = -d_2e^{-a_3},\ d_7 = ma_1d_4 + a_2m, d_8 = a_4 + a_3d_5, d_9 = a_3d_6 - ma_1d_3,$$
$$d_{10} = 1+d_4,\ d_{11} = 1+d_5,\ d_{12} = d_2+d_6-d_1-d_3,\ d_{13} = d_7d_{11}-d_8d_{10},\ d_{14} = d_9d_{11}-d_{12}d_8$$
$$c_2 = \frac{d_{14}}{d_{13}},\ c_4 = \frac{c_2d_{10}-d_{12}}{d_{11}}, c_3 = d_5c_4+d_6,\ c_1 = c_2d_4+d_3,\ b_1 = e^{-e_1}\cos f_1,\ b_2 = e^{-e_1}\sin f_1,$$
$$b_3 = e^{e_6}\cos f_2,\ b_4 = e^{e_6}\sin f_2,\ b_5 = e_1m, b_6 = f_1m, b_9 = b_7 - b_8,\ b_{13} = a_2m,\ b_{12} = a_1m$$

$$b_7 = \frac{b_5 b_2 + b_1 b_6}{b_2}, \quad b_8 = \frac{e_6 b_4 - f_2 b_3}{b_4}, \quad , \quad b_{10} = \frac{b_6}{b_2 b_9}, \quad b_{11} = \frac{f_2}{b_4}, \quad m_1 = a_1 + a_2,$$

$$R_1 = e_2\left[b_{10}e^{-a_1} - b_{12} + b_8\right] + e_4\left[b_{10}e^{-a_2} - b_{13} + b_8\right] + e_7\left[-e^{a_3}b_{11} + a_3 - b_8\right] + e_9\left[a_4 - b_{11}e^{a_4} - b_8\right]$$

$$R_2 = e_3\left[b_{10}e^{-a_1} - b_{12} + b_8\right] + e_5\left[b_{10}e^{-a_2} - b_{13} + b_8\right] + e_8\left[-b_{11}e^{a_3} + a_3 - b_8\right] + e_{10}\left[a_4 - b_{11}e^{a_4} - b_8\right]$$

$$R_3 = R_1 + e_2 + e_4 - e_7 - e_9, \quad R_4 = R_2 + e_3 + e_5 - e_8 - e_{10},$$

$$R_5 = \frac{b_1 R_1 - e_2 e^{-a_1} - e^{-a_2} e_4}{b_2}, \quad R_6 = \frac{b_1 R_2 - e_3 e^{-a_1} - e_5 e^{-a_2}}{b_2}, \quad R_7 = \frac{-\left[b_3 R_3 + e_7 e^{a_3} + e_9 e^{a_4}\right]}{b_4},$$

$$R_8 = \frac{-\left[b_3 R_4 + e_8 e^{a_3} + e_{10} e^{a_4}\right]}{b_4}, t_1 = \frac{-\Pr}{k}, \; t_2 = Ec\; c_1^2\left[m a_1^2 - \sigma_1^2\right], t_3 = Ec\; c_2^2\left[m a_2^2 - \sigma_1^{\,2}\right],$$

$$m_1 = a_1 + a_2, \quad t_4 = 2Ec\; c_1 c_2\left[m a_1 a_2 - \sigma_1^2\right], t_5 = -2\sigma_1^{\,2}\, Ec\, d_1\, c_1, \quad t_6 = -2\sigma_1^{\,2}\, Ec\; c_2\; d_1,$$

$$t_7 = -\sigma_1^2\; Ec\, d_1^2, \; t_8 = \frac{t_1 t_2}{2a_1 t_1 - 4a_1^2}, \quad t_9 = \frac{t_1 t_3}{2a_2 t_1 - 4a_2^2}\; t_{10} = \frac{t_1 t_4}{m_1 t_1 - m_1^2}\;, t_{11} = \frac{t_1 t_5}{a_1 t_1 - a_1^2},$$

$$t_{12} = \frac{t_1 t_6}{a_2 t_1 - a_2^2}, \; t_{13} = \Pr, t_{14} = \frac{t_{13}\; Ec\; a_3\, c_3}{a_3^2 + t_{13} a_3}, \quad t_{15} = \frac{t_{13}\; Ec\; a_4\, c_4}{a_4^2 + a_4 t_{13}}, \; p_2 = -t_{14}e^{a_3} - t_{15}e^{a_4}$$

$$p_1 = 1 + t_7 - t_8 e^{-2a_1} - t_9 e^{-2a_2} - t_{10} e^{-m_1} - t_{11} e^{-a_1} - t_{12} e^{-a_2}, \; p_3 = t_{14} + t_{15} - t_8 - t_9 - t_{10} - t_{11} - t_{12},$$

$$p_4 = a_3 t_{14} + a_4 t_{15} - k\left[t_7 + 2a_1 t_8 + 2a_2 t_9 + m_1 t_{10} + a_1 t_{11} + a_2 t_{12}\right], \; p_5 = e^{-t_1}, \; p_6 = e^{t_{13}}, \; p_7 = k t_1,$$

$$p_8 = 1 - p_5, \quad p_9 = p_6 - 1, \quad p_{10} = p_2 + p_3 - p_1, \quad p_{11} = p_7 p_9 + p_8 p_{13}, \; p_{12} = p_4 p_9 + p_{10} t_{13}$$

$$c_{12} = \frac{p_{12}}{p_{11}}, \; c_{11} = p_1 - p_5 c_{12}, \; c_{13} = p_2 - p_6 c_{14}, \; c_{14} = \frac{p_7 c_{12} - p_4}{t_{13}}, t_{16} = \frac{k}{\Pr}, r_3 = \sqrt{1 + 16\omega^2 t_{16}^2}$$

$$\theta_3 = Tan^{-1}\theta\left[4\omega t_{16}\right], \; f_4 = \frac{-1 + \sqrt{r_3}\cos\left(\frac{\theta_3}{2}\right)}{2t_{16}}, \quad f_5 = \frac{\sqrt{r_3}\left[\sin\left(\frac{\theta_3}{2}\right)\right]}{2t_{16}}, \quad a_{11} = a_1 c_1\left[\alpha_1 e_1 + \alpha_2 f_1\right],$$

$$a_{12} = a_1 c_1\left[\alpha_2 e_1 - \alpha_1 f_1\right], a_{13} = a_2 c_2\left[\alpha_1 e_1 + \alpha_2 f_1\right], a_{14} = a_2 c_2\left[\alpha_2 e_1 - \alpha_1 f_1\right], a_{15} = a_1 a_2\left[c_1 e_4 + c_2 e_2\right]$$

$$a_{16} = a_1 a_2\left[c_1 e_5 + e_3 c_2\right], \; a_{17} = a_1^2 c_1 e_2, \; a_{18} = a_1^2 c_1 e_3, a_{19} = a_2^2 c_2 e_4, a_{20} = a_2^2 c_2 e_5, \; a_{21} = a_1 + e_1,$$

$$a_{22} = a_1 + a_2 = m_1, \; a_{23} = a_2 + e_1, \; g_1 = \frac{A t_1^2 c_{12}\left[t_1 t_{16} + 1\right]}{\left[t_{16} t_1^2 + t_1\right]^2 + \omega^2}, \; h_1 = \frac{A t_1 c_{12}\omega}{\left[t_{16} t_1^2 + t_1\right]^2 + \omega^2},$$

$$g_2 = \frac{A\left[8a_1^3 t_8 t_{16} + 4a_1^2 t_8\right]}{\left[4t_{16}a_1^2 + 2a_1\right]^2 + \omega^2}, \; h_2 = \frac{2a_1 t_8 \omega A}{\left[4t_{16}a_1^2 + 2a_1\right]^2 + \omega^2}, \quad g_3 = \frac{A\left[8a_2^3 t_9 t_{16} + 4a_2^2 t_9\right]}{\left[4t_{16}a_1^2 + 2a_1\right]^2 + \omega^2},$$

$$h_3 = \frac{2a_2 t_9 \omega A}{\left[4t_{16}a_1^2 + 2a_1\right]^2 + \omega^2}, \; g_4 = \frac{A\left[m_1^3 t_{10} t_{16} + m_1^2 t_{10}\right]}{\left[t_{16} m_1^2 + m_1\right]^2 + \omega^2}, \; h_4 = \frac{A m_1 t_{10}\omega}{\left[t_{16} m_1^2 + m_1\right]^2 + \omega^2},$$

$$g_5 = \frac{A\left[a_1^3 t_1 t_{16} + a_1^2 t_1\right]}{\left[t_{16}a_1^2 + a_1\right]^2 + \omega^2}, \; h_5 = \frac{A t_1 a_1 \omega}{\left[t_{16}a_1^2 + a_1\right]^2 + \omega^2}, \quad g_6 = \frac{A\left[a_2^3 t_{12} t_{16} + a_2^2 t_{12}\right]}{\left[t_{16}a_2^2 + a_2\right]^2 + \omega^2},$$

$$h_6 = \frac{A t_{12} a_2 \omega}{\left[t_{16}a_2^2 + a_2\right]^2 + \omega^2}, \; r_{11} = 2\,m\,Ec\left[1 + 2a_{21}t_{16}\right], r_{12} = 2\,m\,Ec\,t_{16}\left[a_{21}^2 - f_1^{\,2}\right], \; r_{13} = 2\,m\,Ec\,\omega,$$

$$r_{14} = t_{16}^2\left[a_{21}^2 - f_1^2\right]^2 - \left[1+2a_{21}t_{16}\right] + \omega^2, \quad r_{15} = 2t_{16}\left[a_{21}^2 - f_1^2\right]\left[1+2a_{21}t_{16}\right],$$

$$r_{16} = \frac{a_{11}\left[r_{11}r_{15}f_1^2 + r_{12}r_{14}\right] + a_{12}f_1\left[r_{11}r_{14} - r_{12}r_{15}\right]}{r_{15}^2 + r_{14}^2}, \quad r_{17} = \frac{a_{12}\left[r_{11}r_{15}f_1^2 + r_{12}r_{14}\right] + a_{11}f_1\left[r_{12}r_{15} - r_{11}r_{14}\right]}{r_{15}^2 + r_{14}^2}$$

$$r_{18} = \frac{a_{12}f_1r_{13}r_{15} - a_{11}r_{13}r_{14}}{r_{15}^2 + r_{14}^2}, \quad r_{19} = \frac{-\left(a_{11}f_1r_{13}r_{15} + a_{12}r_{13}r_{14}\right)}{r_{15}^2 + r_{14}^2}, \quad r_{21} = 2\,m\,Ec\left[1+2a_{23}t_{16}\right],$$

$$r_{22} = 2\,m\,Ec\,t_{16}\left[a_{23}^2 - f_1^2\right], \quad r_{23} = 2\,m\,Ec\,\omega, \quad r_{24} = t_{16}^2\left[a_{23}^2 - f_1^2\right]^2 - \left[1+2a_{23}t_{16}\right]^2 + \omega^2,$$

$$r_{25} = 2t_{16}\left[a_{23}^2 - f_1^2\right]\left[1+2a_{23}t_{16}\right], \quad r_{26} = \frac{a_{13}\left[r_{21}r_{25}f_1^2 + r_{24}r_{22}\right] + a_{14}f_1\left[r_{21}r_{24} - r_{22}r_{25}\right]}{r_{25}^2 + r_{24}^2},$$

$$r_{27} = \frac{a_{14}\left[r_{21}r_{25}f_1^2 + r_{24}r_{22}\right] + a_{13}f_1\left[r_{22}r_{25} - r_{21}r_{24}\right]}{r_{25}^2 + r_{24}^2}, \quad r_{28} = \frac{a_{14}f_1r_{23}r_{25} - a_{13}r_{23}r_{24}}{r_{25}^2 + r_{24}^2},$$

$$r_{29} = \frac{-\left[a_{13}f_1r_{23}r_{25} + a_{14}r_{23}r_{24}\right]}{r_{25}^2 + r_{24}^2}, \quad g_7 = \frac{2\,m\,Ec\,a_{15}\left[t_{16}a_{22}^2 + a_{22}\right]}{\left(t_{16}a_{22}^2 + a_{22}\right)^2 + \omega^2}, \quad h_7 = \frac{2\,m\,Ec\,a_{15}\,\omega}{\left(t_{16}a_{22}^2 + a_{22}\right)^2 + \omega^2},$$

$$g_8 = \frac{2\,m\,Ec\,a_{17}a_1\left[4t_{16}a_1 + 2\right]}{\left[4t_{16}a_1^2 + 2a_1\right]^2 + \omega^2}, \quad h_8 = \frac{2\,m\,Ec\,a_{17}\,\omega}{\left[4t_{16}a_1^2 + 2a_1\right]^2 + \omega^2}, \quad g_9 = \frac{2\,m\,Ec\,a_{19}\left[4t_{16}a_2^2 + 2a_2\right]}{\left[4t_{16}a_2^2 + 2a_2\right]^2 + \omega^2},$$

$$h_9 = \frac{2\,m\,Ec\,a_{19}\,\omega}{\left[4t_{16}a_2^2 + 2a_2\right]^2 + \omega^2}, \quad g_{10} = \frac{-2\,m\,Ec\,a_{16}\,\omega}{\left[t_{16}a_{22}^2 + a_{22}\right]^2 + \omega^2}, \quad h_{10} = \frac{2\,m\,Ec\,a_{16}\left[t_{16}a_{22}^2 + a_{22}\right]}{\left[t_{16}a_{22}^2 + a_{22}\right]^2 + \omega^2},$$

$$g_{11} = \frac{-2\,m\,Ec\,a_{18}\,\omega}{\left[4t_{16}a_1^2 + 2a_1\right]^2 + \omega^2}, \quad h_{11} = \frac{2\,m\,Ec\,a_{18}\left[4t_{16}a_1^2 + 2a_1\right]}{\left[4t_{16}a_1^2 + 2a_1\right]^2 + \omega^2}, \quad g_{12} = \frac{-2\,m\,Ec\,a_{20}\,\omega}{\left[4t_{16}a_2^2 + 2a_2\right]^2 + \omega^2},$$

$$h_{12} = \frac{2\,m\,Ec\,a_{20}\left[4t_{16}a_2^2 + 2a_2\right]}{\left[4t_{16}a_2^2 + 2a_2\right]^2 + \omega^2}, \quad r_{31} = -2\sigma_1^2Ec\left[1+2a_{21}t_{16}\right], \quad r_{32} = -2\sigma_1^2Ec\,t_{16}\left[a_{21}^2 - f_1^2\right],$$

$$r_{33} = -2\sigma_1^2Ec\,\omega, \quad r_{34} = t_{16}^2\left[a_{21}^2 - f_1^2\right]^2 - \left[1+2a_{21}^2t_{16}\right]^2 + \omega^2,$$

$$r_{35} = 2t_{16}\left[a_{21}^2 - f_1^2\right]\left[1+2a_{21}t_{16}\right], \quad r_{36} = \frac{c_1\alpha_1\left[r_{31}r_{35}f_1^2 + r_{32}r_{34}\right] + c_1\alpha_2f_1\left[r_{31}r_{34} - r_{32}r_{35}\right]}{r_{35}^2 + r_{34}^2}$$

$$r_{38} = \frac{c_1\alpha_2f_1r_{35}r_{33} - c_1\alpha_1r_{33}r_{34}}{r_{35}^2 + r_{34}^2} \quad r_{37} = \frac{c_1\alpha_2\left[r_{31}r_{35}f_1^2 + r_{32}r_{34}\right] + c_1\alpha_2f_1\left[r_{32}r_{35} - r_{31}r_{34}\right]}{r_{35}^2 + r_{34}^2},$$

$$r_{39} = \frac{-c_1r_{33}\left[\alpha_1f_1r_{35} - \alpha_2r_{34}\right]}{r_{35}^2 + r_{34}^2}, \quad r_{43} = -2\sigma_1^2Ec\,\omega \quad r_{41} = -2\sigma_1^2Ec\left[1+2a_{23}t_{16}\right]$$

$$r_{42} = -2\sigma_1^2Ec\,t_{16}\left[a_{23}^2 - f_1^2\right] \quad r_{44} = t_{16}^2\left[a_{23}^2 - f_1^2\right] - \left[1+2a_{23}^2t_{16}\right]^2 + \omega^2,$$

$$r_{45} = 2t_{16}\left[a_{23}^2 - f_1^2\right]\left[1+2a_{23}^2t_{16}\right], \quad r_{46} = \frac{c_2\alpha_1\left[r_{41}r_{45}f_1^2 + r_{42}r_{44}\right] + c_2\alpha_2f_1\left[r_{41}r_{44} - r_{42}r_{45}\right]}{r_{45}^2 + r_{44}^2},$$

$$r_{47}=\frac{c_2\alpha_2\left[r_{41}r_{45}f_1^2+r_{42}r_{44}\right]+c_2\alpha_1 f_1\left[r_{42}r_{45}-r_{41}r_{44}\right]}{r_{45}^2+r_{44}^2},\ r_{48}=\frac{c_2r_{43}\left[\alpha_2 f_1r_{45}-\alpha_1r_{44}\right]}{r_{45}^2+r_{44}^2},$$

$$r_{49}=\frac{-c_2r_{43}\left[\alpha_1 f_1r_{45}+\alpha_2r_{44}\right]}{r_{45}^2+r_{44}^2},r_{51}=-2\sigma_1^2\,Ec\left[1+2e_1t_{16}\right],\qquad r_{52}=-2\sigma_1^2\,Ec\,t_{16}\left[e_1^2-f_1^2\right],$$

$$r_{53}=-2\sigma_1^2Ec\,\omega,\quad r_{54}=t_{16}^2\left[e_1^2-f_1^2\right]^2-\left[1+2e_1^2t_{16}\right]^2+\omega^2,\quad r_{55}=2t_{16}\left[e_1^2-f_1^2\right]\left[1+2e_1^2t_{16}\right]^2,$$

$$r_{56}=\frac{\alpha_1d_1\left[r_{51}r_{55}f_1^2+r_{52}r_{54}\right]+\alpha_2d_1f_1\left[r_{51}r_{54}-r_{52}r_{55}\right]}{r_{55}^2+r_{54}^2},r_{58}=\frac{d_1r_{53}\left[\alpha_2 f_1r_{55}-\alpha_1r_{54}\right]}{r_{54}^2+r_{55}^2}$$

$$r_{57}=\frac{\alpha_2d_1\left[r_{51}r_{55}f_1^2+r_{52}r_{54}\right]+\alpha_1d_1f_1\left[r_{52}r_{55}-r_{51}r_{54}\right]}{r_{55}^2+r_{54}^2}\qquad r_{59}=\frac{-d_1r_{53}\left[\alpha_1 f_1r_{55}+\alpha_2r_{54}\right]}{r_{55}^2+r_{54}^2}$$

$$g_{13}=\frac{-2\sigma_1{}^2Ec\,\left\{\left(t_{16}a_{22}^2+a_{22}\right)\left(c_1e_4+c_2e_2\right)-\omega\left(c_1e_5+c_2e_3\right)\right\}}{\left[t_{16}a_{22}^2+a_{22}\right]^2+\omega^2}$$

$$h_{13}=\frac{-2\sigma_1{}^2Ec\,\left\{\left(t_{16}a_{22}^2+a_{22}\right)\left(c_1e_5+c_2e_3\right)+\omega\left(c_1e_4+c_2e_2\right)\right\}}{\left[t_{16}a_{22}^2+a_{22}\right]^2+\omega^2}$$

$$g_{14}=\frac{-2\sigma_1{}^2Ec\,\left\{\left(4t_{16}a_1^2+2a_1\right)c_1e_2-\omega c_1e_3\right\}}{\left[4t_{16}a_1^2+2a_1\right]^2+\omega^2},\quad h_{14}=\frac{-2\sigma_1^2Ec\,\left\{c_1e_3\left(4t_{16}a_1^2+2a_1\right)+\omega c_1e_2\right\}}{\left[4t_{16}a_1^2+2a_1\right]^2+\omega^2},$$

$$g_{15}=\frac{-2\sigma_1^2Ec\left\{c_2e_4\left(4t_{16}a_2^2+2a_2\right)-\omega c_2e_5\right\}}{\left[4t_{16}a_2^2+2a_2\right]^2+\omega^2},\quad h_{15}=\frac{-2\sigma_1^2Ec\,\left\{\left(4t_{16}a_2^2+2a_2\right)c_2e_5+\omega c_2e_4\right\}}{\left[4t_{16}a_2^2+2a_2\right]^2+\omega^2}$$

$$g_{16}=\frac{-2\sigma_1^2Ec\left\{\left(t_{16}a_1^2+a_1\right)d_1e_2-\omega d_1e_2\right\}}{\left[t_{16}a_1^2+a_1\right]^2+\omega^2},\qquad h_{16}=\frac{-2\sigma_1^2Ec\left\{d_1e_3\left(t_{16}a_1^2+a_1\right)+\omega d_1e_2\right\}}{\left[t_{16}a_1^2+a_1\right]^2+\omega^2},$$

$$g_{17}=\frac{-2\sigma_1^2Ec\,\left\{d_1e_4\left(t_{16}a_2^2+a_2\right)-\omega d_1e_5\right\}}{\left[t_{16}a_2^2+a_2\right]^2+\omega^2},\qquad h_{17}=\frac{-2\sigma^2Ec\left\{d_1e_5\left(t_{16}a_2^2+a_2\right)+\omega d_1e_4\right\}}{\left[t_{16}a_2^2+a_2\right]^2+\omega^2},$$

$$r_4=\sqrt{1+16B_2^2\omega^2},\quad \theta_4=Tan^{-1}\left(4B_2\omega\right),\qquad f_6=\frac{-1+\sqrt{r_4}\cos\left(\frac{\theta_4}{2}\right)}{2B_2},\qquad f_7=\frac{\sqrt{r_4}\sin\left(\frac{\theta_4}{2}\right)}{2B_2},$$

$$f_8=a_3+e_6,\qquad f_9=a_4+e_6,\qquad f_{10}=a_3+a_4,\qquad g_{20}=\frac{-At_{13}c_{14}\left(B_2t_{13}^2+t_{13}\right)}{\left(B_2t_{13}^2+t_{13}\right)^2+\omega^2}$$

$$h_{20}=\frac{-A\omega t_{13}c_{14}}{\left(B_2t_{13}^2+t_{13}\right)^2+\omega^2},\qquad g_{21}=\frac{-At_{14}a_3\left(B_2a_3^2+a_{13}\right)}{\left(B_2a_3^2+a_3\right)^2+\omega^2},\qquad h_{21}=\frac{-A\omega t_{14}a_3}{\left(B_2a_3^2+a_3\right)^2+\omega^2},$$

$$g_{22} = \frac{-At_{15}a_4\left(B_2a_4^2 + a_4\right)}{\left(B_2a_4^2 + a_4\right)^2 + \omega^2}, \quad h_{22} = \frac{-At_{15}a_4\omega}{\left(B_2a_4^2 + a_4\right)^2 + \omega^2}, \quad l_1 = 2\,Ec\,a_3\,c_3\,e_6\left(\alpha_3 - \alpha_4 f_2\right),$$

$$l_2 = 2\,Ec\,a_3\,c_3\,e_6\left(\alpha_4 - \alpha_3 f_2\right), \quad l_3 = B_2 f_8^2 - B_2 f_2^2 + f_8, \quad l_4 = 2B_2 f_8 + 1, \quad l_6 = 2l_3l_4,$$

$$l_5 = l_3^2 - l_4^2 f_2^2 + \omega^2, \quad l_7 = \frac{l_1l_4l_6 f_2^2 - l_2l_3l_6 f_2 + l_2l_4l_5 f_2 + l_1l_3l_5}{\left[l_6^2 f_2^2 + l_5^2\right]} \quad l_9 = \frac{\omega l_1l_5 - l_2 f_2 l_6\omega}{l_6^2 f_2^2 + l_5^2}$$

$$l_8 = \frac{l_2l_4l_6 f_2^2 - l_1l_4l_2l_5 + l_1l_2l_3l_6 + l_2l_3l_5}{\left[l_6^2 f_2^2 + l_5^2\right]}, \quad l_{10} = \frac{l_1l_6 f_2\omega + l_2l_5\omega}{l_6^2 f_2^2 + l_5^2}, \quad l_{11} = 2E_c a_4c_4e_6\left(\alpha_3 - \alpha_4 f_2\right),$$

$$l_{16} = 2l_{13}l_{14}, \quad l_{12} = 2Ec\,a_4c_4e_6\left(\alpha_4 - \alpha_3 f_2\right), \quad l_{13} = B_2 f_9^2 - B_2 f_2^2 + f_9, \quad l_{14} = 2B_2 f_9 + 1,$$

$$l_{15} = l_{13}^2 - l_{14}^2 f_2^2 + \omega^2$$

$$l_{17} = \frac{l_{11}l_{14}l_{16} f_2^2 - l_{12}l_{13}l_{16} f_2 + l_{12}l_{14}l_{15} f_2 + l_{11}l_{13}l_{15}}{l_{16}^2 f_2^2 + f_{15}^2}, \quad l_{18} = \frac{l_{12}l_{14}l_{16} f_2^2 - l_{11}l_{14}l_{15} f_2 + l_{11}l_{13}l_{16} f_2 + l_{12}l_{13}l_{15}}{l_{16}^2 f_2^2 + f_{15}^2},$$

$$l_{19} = \frac{\omega l_{11}l_{15} - l_{12}l_{16} f_2\omega}{l_{16}^2 f_2^2 + l_{15}^2}, \quad l_{20} = \frac{l_{11}l_{16} f_2\omega + l_{12}l_{15}\omega}{l_{16}^2 f_2^2 + l_{15}^2} \quad g_{23} = \frac{8a_3^4c_3e_7Ec\,B_2 + 4a_3^3c_3e_7Ec - 2a_3^3c_3e_8Ec\,\omega}{\left(4B_2a_3^2 + 2a_3\right)^2 + \omega^2}$$

$$h_{23} = \frac{8a_3^4c_3e_8EcB_2 + 4a_3^3c_3e_8Ec + 2a_3^3c_3e_7Ec\omega}{\left(4B_2a_3^2 + 2a_3\right)^2 + \omega^2} \quad g_{24} = \frac{8a_4^4c_9B_2Ec + 4a_4^3c_4e_9B_2Ec - 2a_4^3c_4e_{10}\omega Ec}{\left(4B_2a_4^2 + 2a_4\right)^2 + \omega^2}$$

$$h_{24} = \frac{8a_4^4c_4e_{10}B_2Ec + 4a_4^2c_4e_{10}B_2Ec + 2a_4^3c_4e_9\omega Ec}{\left(4B_2a_4^2 + 2a_4\right)^2 + \omega^2}, \quad x_1 = a_3a_4\left[c_3e_9 + c_4e_7\right],$$

$$x_2 = a_3a_4\left[c_3e_{10} + c_4e_8\right], \; g_{25} = \frac{x_1\left(f_{10} + f_{10}^2B_2\right) - x_2\omega}{\left[B_2 f_{10}^2 + f_{10}\right]^2 + \omega^2}, \quad h_{25} = \frac{x_1\omega + x_2\left(f_{10} + f_{10}^2B_2\right)}{\left[B_2 f_{10}^2 + f_{10}\right]^2 + \omega^2},$$

$$R_{11} = -e^{f_4}\left\{\begin{array}{l} e^{-a_{21}}\left[\left(r_{16} + r_{36}\right)\cos f_1 - \left(r_{17} + r_{37}\right)\sin f_1\right] + e^{-a_{23}}\left[\left(r_{26} + r_{46}\right)\cos f_1 - \left(r_{27} + r_{47}\right)\sin f_1\right] \\ + e^{-e_1}\left[r_{56}\cos f_1 - r_{57}\sin f_1\right] + e^{-a_1}\left[g_5 + g_{16}\right] + e^{-a_2}\left[g_6 + g_{17}\right] + e^{-2a_2}\left[g_3 + g_9 + g_{12} + g_{15}\right] \\ + e^{-t_1}g_1 + e^{-m_1}\left[g_7 + g_{10} + g_{13} + g_4\right] \end{array}\right\},$$

$$R_{12} = e^{f_4}\left\{\begin{array}{l} e^{-a_{21}}\left[\left(r_{18} + r_{38}\right)\cos f_1 - \left(r_{19} + r_{39}\right)\sin f_1\right] + e^{-a_{23}}\left[\left(r_{28} + r_{48}\right)\cos f_1 - \left(r_{29} + r_{49}\right)\sin f_1\right] \\ + e^{-e_1}\left[\left(r_{58}\right)\cos f_1 - \left(r_{59}\right)\sin f_1\right] + e^{-a_1}\left[h_5 + h_{16}\right] - e^{-a_2}\left[h_6 + h_{17}\right] - e^{-2a_1}\left[h_2 + h_8 + h_{11} + h_{14}\right] \\ - e^{-2a_2}\left[h_3 + h_9 + h_{12} + h_{15}\right] - e^{-m_1}\left[h_4 + h_7 + h_{10} + h_{13}\right] - h_1e^{-t_1} - \dfrac{At_7}{\omega} \end{array}\right\}$$

$$R_{13} = -e^{-f_6}\left\{\begin{array}{l} e^{f_8}\left[l_7\cos f_2 + l_8\sin f_2\right] + e^{f_9}\left[l_{17}\cos f_2 + l_{18}\sin f_2\right] + g_{20}e^{t_{13}} + g_{21}e^{a_3} \\ . + g_{22}e^{a_4} + g_{23}e^{2a_3} + g_{24}e^{2a_4} + g_{25}e^{f_{10}} \end{array}\right\}$$

$$R_{14} = -e^{-f_6}\left\{\begin{array}{l} e^{f_8}\left[l_9\cos f_2 + l_{10}\sin f_2\right] + e^{f_9}\left[l_{19}\cos f_2 + l_{20}\sin f_2\right] + h_{20}e^{t_{13}} + h_{21}e^{a_3} + h_{22}e^{a_4} \\ + h_{23}e^{2a_3} + h_{24}e^{2a_4} + h_{25}e^{f_{10}} \end{array}\right\},$$

$$R_{15}=\left[l_7+l_{17}+g_{20}+g_{21}+g_{22}+g_{23}+g_{24}+g_{25}\right]$$
$$-\left\{\begin{array}{l}g_1+g_2+g_3+g_4+g_5+g_6+g_7+g_8+g_9+g_{10}+g_{11}+g_{12}+g_{13}+g_{15}+g_{15}\\+g_{16}+g_{17}+r_{16}+r_{26}+r_{36}+r_{46}\end{array}\right\},$$

$$R_{16}=\left[l_9+l_{19}+h_{20}+h_{21}+h_{22}+h_{23}+h_{24}+h_{25}\right]$$
$$-\left\{\begin{array}{l}-r_{18}-r_{38}-r_{28}-r_{48}-r_{58}+h_5+h_{16}+h_6+h_{17}+h_2+h_{14}+h_{11}+h_8+h_3+h_9+h_{12}+h_{15}\\+h_4+h_7+h_{10}+h_{13}+h_1+\dfrac{At_7}{\omega}\end{array}\right\},$$

$$R_{17}=f_2l_8+f_2l_9+f_2l_{18}+f_9l_{17}+t_{13}g_{20}+a_3g_{21}+a_4g_{22}+2a_3g_{23}+g_{24}+f_{10}g_{20}$$
$$-K\left\{\begin{array}{l}f_1\left[r_{17}+r_{37}\right]+a_{21}\left[r_{16}+r_{36}\right]+f_1r_{57}+e_1r_{56}+a_1\left(g_5+g_{16}\right)+a_2\left(g_6+g_{17}\right)+2a_1\left(g_2+g_8+g_{11}+g_{14}\right)\\+2a_2\left(g_3+g_9+g_{12}+g_{15}\right)+t_1g_1+m_1\left(g_4+g_7+g_{10}+g_{13}\right)\end{array}\right\}$$

$$R_{18}=l_{10}f_2+f_8l_9+l_{20}f_2+l_9f_{19}+t_{13}h_{20}+a_3h_{21}+a_4h_{22}+2a_3h_{23}+2a_4h_{24}+f_{10}h_{25}$$
$$+k\left\{\begin{array}{l}f_1\left(r_{19}+r_{39}\right)+a_{21}\left(r_{18}+r_{38}\right)+f_1\left(r_{29}+r_{49}\right)+a_{23}\left(r_{28}+r_{48}\right)+f_1r_{59}+e_1r_{58}-a_1\left(h_5+h_{16}\right)-a_2\left(h_6+h_{17}\right)\\-2a_1\left(h_2+h_8+h_{11}+h_{14}\right)-t_1h_1-2a_2\left(h_3+h_9+h_{12}+h_{15}\right)-m_1\left(h_4+h_7+h_{10}+h_{13}\right)\end{array}\right\}$$

$$l_{21}=\frac{kf_5\cos f_5}{\sin f_5}+kf_4,\qquad l_{22}=\frac{f_7\cos f_7}{\sin f_7}-f_6,\qquad R_{19}=\frac{kf_5R_{11}}{\sin f_5}+\frac{R_{13}f_7}{\sin f_7}+R_{17}$$

$$R_{20}=\frac{kf_5R_{12}}{\sin f_5}+\frac{R_{14}f_7}{\sin f_2}+R_{18},\qquad R_{21}=\frac{l_{22}R_{15}+R_{19}}{l_{22}+l_{21}},\qquad R_{22}=\frac{l_{22}R_{16}+R_{20}}{l_{22}+l_{21}},\qquad R_{23}=R_{21}-R_{15},$$

$$R_{24}=R_{22}-R_{16},\qquad R_{25}=\frac{R_{21}\cos f_5-R_{11}}{\sin f_5},\qquad R_{26}=\frac{R_{22}\cos f_5-R_{12}}{\sin f_5},$$

## 5. RATE OF HEAT TRANSFER

The rate of heat transfer (Nusselt number) through the channel wall to the fluid is given by

$$Nu=\left[\frac{d\theta}{dy}\right] \tag{41}$$

Based on the analytical solutions reported above the rate of heat transfer at the bottom wall is given by

$$Nu_1=\left[\frac{d\theta_1}{dy}\right]_{y=-1} \tag{42}$$

$$Nu_1 = t_1c_{12}e^{-t_1} + t_7 + t_8 2a_1 e^{-2a_1} + t_9 2a_2 + t_{10}e^{-m_1} + t_{11}a_1e^{-a_1} + t_{12}a_2e^{-a_2} +$$

$$\varepsilon\cos\omega t\left\{\begin{array}{l} f_4e^{-f_4}\left[\alpha_{11}\cos f_5 - \alpha_{12}\sin f_5\right] + f_5e^{-f_4}\left[\alpha_{11}\sin f_5 + \alpha_{12}\cos f_5\right] \\ +a_{21}e^{-a_{21}}\left[(r_{16}+r_{36})\cos f_1 - (r_{17}+r_{37})\sin f_1\right] + f_1e^{-a_{21}}\left[(r_{16}+r_{36})\sin f_1 + (r_{17}+r_{37})\cos f_1\right] \\ +a_{23}e^{-a_{23}}\left[(r_{26}+r_{46})\cos f_1 - (r_{27}+r_{47})\sin f_1\right] + f_1e^{-a_{23}}\left[(r_{26}+r_{46})\sin f_1 + (r_{27}+r_{47})\cos f_1\right] \\ +e_1e^{-e_1}[r_{56}\cos f_1 - r_{57}\sin f_1] + e^{-e_1} f_1[r_{56}\sin f_1 + r_{57}\cos f_1] + a_1e^{-a_1}[g_5+g_{16}] + a_2e^{-a_2}[g_6+g_{17}] \\ +2a_1e^{-2a_1}[g_2+g_8+g_{11}+g_{14}] + 2a_2e^{-2a_2}[g_3+g_9+g_{12}+g_{15}] + t_1g_1e^{-t_1} + a_{22}e^{-a_{22}}[g_4+g_7+g_{10}+g_{13}] \end{array}\right\}$$

$$-\varepsilon\sin\omega t\left\{\begin{array}{l} -a_2e^{-a_2}[(r_{18}+r_{38})\cos f_1 - (r_{19}+r_{39})\sin f_1] - f_1e^{-a_2}[(r_{18}+r_{38})\sin f_1 + (r_{19}+r_{39})\cos f_1] \\ +a_{23}e^{-a_{23}}\left[(r_{28}+r_{48})\cos f_1 - (r_{29}+r_{49})\sin f_1\right] - f_1e^{-a_{23}}\left[(r_{28}+r_{48})\sin f_1 + (r_{29}+r_{49})\cos f_1\right] \\ -e_1e^{-e_1}[r_{58}\cos f_1 - r_{59}\sin f_1] - e^{-e_1} f_1[r_{58}\sin f_1 + r_{59}\cos f_1] + a_1e^{-a_1}[h_5+h_{16}] + a_2e^{-a_2}[h_6+h_{17}] \\ +2a_1e^{-2a_1}[h_2+h_8+h_{11}+h_{14}] + 2a_2e^{-2a_2}[h_3+h_9+h_{12}+h_{15}] + t_1g_1e^{-t_1} + a_{22}e^{-a_{22}}[h_4+h_7+h_{10}+h_{13}] \end{array}\right\}$$

(43)

At the top wall, it is given by

$$Nu_2 = \left[\frac{d\theta_2}{dy}\right]_{y=1} \qquad (44)$$

$$= t_{13}C_{14}e^{t_{13}} + a_3t_{14}e^{a_3} + a_4t_{15}e^{a_4}$$

$$+\varepsilon\cos\omega t\left\{\begin{array}{l} f_6e^{f_6}[\alpha_{13}\cos f_7 + \alpha_{14}\sin f_7] + f_7e^{f_6}[\alpha_{13}\sin f_7 + \alpha_{14}\cos f_7] \\ + f_8e^{f_8}[l_7\cos f_2 + l_8\sin f_2] + f_7e^{f_8}[-l_7\sin f_2 + l_8\cos f_2] + f_9e^{f_9}[l_{17}\cos f_2 + l_{18}\sin f_2] \\ + f_2e^{f_9}[-l_{17}\sin f_2 + l_{18}\cos f_2] + g_{20}t_{13}e^{t_{13}} + g_{21}a_3e^{a_3} + g_{22}a_4e^{a_4} + g_{23}2a_3e^{2a_3} + g_{24}2a_4e^{2a_4} + g_{25}f_{10}e^{f_{10}} \end{array}\right\}$$

$$-\varepsilon\sin\omega t\left\{\begin{array}{l} f_8e^{f_8}[l_9\cos f_2 + l_{10}\sin f_2] + f_2e^{f_8}[-l_9\sin f_2 + l_{10}\cos f_2] + f_9e^{f_9}[l_{19}\cos f_2 + l_{20}\sin f_2] \\ + f_2e^{f_9}[-l_{19}\sin f_2 + l_{20}\cos f_2] + h_{20}t_{13}e^{t_{13}} + h_{21}a_3e^{a_3} + h_{22}a_4e^{a_4} + h_{23}2a_3e^{2a_3} + h_{24}2a_4e^{2a_4} + h_{25}f_{10}e^{f_{10}} \end{array}\right\}$$

(45)

## 6. MASS FLUX

The dimensionless mass flow rate per unit with of the channel is

$$Q = F_1 + F_2 \qquad (46)$$

where

$$F_1 = \int_{-1}^{0} u_1(y,t)dy \qquad (47)$$

$$F_1 = \frac{C_1}{a_1}[1-e^{-a_1}] + \frac{C_2}{a_2}[1-e^{-a_2}] \quad - \frac{P}{\sigma^2+M_1^2}$$
$$+\varepsilon\left\{\begin{array}{l} \frac{e_1}{e_1^2+f_1^2}\left(R_1\cos\omega t - R_2\sin\omega t\right) + \frac{f_1}{e_1^2+f_1^2}\left(R_6\sin\omega t - R_5\cos\omega t\right) + \frac{e_2\cos\omega t - e_3\sin\omega t}{a_1} \\ + \frac{e_4\cos\omega t - e_5\sin\omega t}{a_2} - \frac{e^{-e_1}}{e_1^2+f_1^2}(e_1\cos f_1 - f_1\sin f_1)(R_1\cos\omega t - R_2\sin\omega t) \\ + \frac{e^{-e_1}}{e_1^2+f_1^2}(e_1\sin f_1 + f_1\cos f_1)(R_5\cos\omega t - R_6\sin\omega t) \end{array}\right\}$$

(48)

And

$$F_2 = \int_0^1 u_2(y,t)dy \qquad (49)$$

$$F_2 = \frac{C_3}{a_3}(e^{a_3}-1) + \frac{C_4}{a_4}(e^{a_4}-1) + d_2$$
$$+\varepsilon\left\{\begin{array}{l} \frac{e^6}{e_6^2+f_2^2}[(e_6\cos\omega t + f_2\sin\omega t)(R_3\cos\omega t - R_4\sin\omega t)] + [(e_6\sin f_2 - f_2\cos f_2)(R_7\cos\omega t - R_8\sin\omega t)] \\ + \frac{e^{a_3}}{a_3}[e_7\cos\omega t - e_8\sin\omega t] + \frac{e^{a_4}}{a_4}[e_9\cos\omega t - e_{10}\sin\omega t] + \cos\omega t\left[\frac{e_6R_3}{e_6^2+f_2^2} - \frac{f_2R_7}{e_6^2+f_2^2} + \frac{e_7}{a_3} + \frac{e_9}{a_4}\right] \\ -\sin\omega t\left[\frac{e_6R_4}{e_6^2+f_2^2} - \frac{f_2R_8}{e_6^2+f_2^2} - \frac{e_8}{a_3} - \frac{e_{10}}{a_4}\right] \end{array}\right\}$$

(50)

## 7. INTERFACE VELOCITY

Taking y = 0 in the equation (37) or in equation (38) we get the interface velocity as

$$u_0 = C_1 + C_2 - \frac{P}{\sigma^2+M_1^2} + \varepsilon\{\cos\omega t[R_1+e_2+e_4] - \sin\omega t[R_2+e_3+e_5]\} \qquad (51)$$

(or)

$$u_0 = C_3 + C_4 + d_2 + \varepsilon\{\cos\omega t[R_3+e_7+e_9] - \sin\omega t[R_4+e_8+e_{10}]\} \qquad (52)$$

## 8 RESULTS AND DISCUSSION

In this chapter, oscillatory flow of conducting viscous fluid in a horizontal composite porous medium channel is investigated. The closed-form solutions are reported for small ε such that oscillation amplitude $\varepsilon A \leq 1$.

Flow solutions are depicted graphically for pressure gradient P, viscosity ratio m, magnetic fields $M_1$ and $M_2$, porous medium parameter $\sigma$ and Prandtl number Pr, conductivity ratio K, and Eckert number Ec, on the velocity and the temperature in both regions of the channel.

The variation of velocity with y is calculated, from equations (29), (33), (30), and (34), for different values of P, $\sigma$, $M_1$, $M_2$ and m is shown in Figures 2, 3, 4, 5 and 6 for fixed A=0.01, $\varepsilon$ = 0.01 and t = 0.01. We observe that the velocity increases with the decreasing P or $\sigma$ or $M_1$ or $M_2$ or m . The velocity increases in Region 1 and after attaining the maximum value decreases in Region 2.

From the equations (31), (32), (35) and (36), we have calculated the temperature as a function of y, for fixed $M_1$ = 1, $M_2$ =2, $\sigma$ = 1, A=0.01, $\varepsilon$ = 0.01, m = 1, Ec =0.01 and t = 0.01 and for different values of Pr and K is shown in Figures 7 and 8. We observe that in Region 1, the temperature increases with decreasing Pr or K from y= -1 to y= -0.1 and in Region 2 the temperature increases with the increasing Pr or K. The temperature attains the maximum value at y= -1, in Region 1 and the temperature decreases with increasing of y values $(-1 \le y \le 1)$ from Region 1 to Region 2.

The variation temperature is evaluated as a function of y, for fixed P = -1, $M_1$ = 1, $M_2$ =2, $\sigma$ = 1, A=0.01, $\varepsilon$ = 0.01, m = 1, K = 1, Pr =0.7 and t = 0.01 and for different values of Eckert number Ec and is shown in Fig. 9. We observe that in Region 1 the temperature increases with decreasing of Eckert number Ec, from y= -1 to y= -0.4 and increases with increasing of Eckert number Ec in the remaining portion of channel. The temperature attains the maximum value at y= -1 in Region 1 and the temperature decreases with the increasing of y value $(-1 \le y \le 1)$ from Region 1 to Region 2.

The variation of temperature with y is calculated, for different values of Region 1 $M_1$ and $M_2$ and is shown in Figures 10 and 11 for fixed P = -1, $\sigma$ = 1, A=0.01, $\varepsilon$ = 0.01, m = 1, K = 1, Ec = 0.01, Pr =0.7 and t = 0.01. We observe that the temperature increases with the decreasing $M_1$ or $M_2$. The temperature increases with the decrease of $M_1$ or $M_2$ in Region 1. It attains the maximum value at y = -1 and the temperature decreases with the increasing of y value $(-1 \le y \le 1)$ from Region 1 to Region 2.

| $m$ | $M_1=0.1$ | $M_1=0.5$ | $M_1=1$ | $M_2=1$ | $M_2=1.5$ | $M_2=2$ | $\sigma=0.01$ | $\sigma=0.1$ | $\sigma=1$ |
|---|---|---|---|---|---|---|---|---|---|
| **2** | 0.215 | 0.2108 | 0.1954 | 0.2608 | 0.2278 | 0.1954 | 0.1954 | 0.1952 | 0.1799 |
| **2.1** | 0.2106 | 0.2002 | 0.1918 | 0.2548 | 0.2231 | 0.1918 | 0.1918 | 0.1917 | 0.1769 |
| **2.2** | 0.2065 | 0.2003 | 0.1883 | 0.249 | 0.2185 | 0.1883 | 0.1883 | 0.1882 | 0.174 |
| **2.3** | 0.2028 | 0.1969 | 0.1849 | 0.2435 | 0.2141 | 0.1849 | 0.1849 | 0.1848 | 0.1711 |
| **2.4** | 0.2208 | 0.1932 | 0.1816 | 0.2382 | 0.2098 | 0.1816 | 0.1816 | 0.1815 | 0.1683 |
| **2.5** | 0.1927 | 0.1896 | 0.1784 | 0.233 | 0.2057 | 0.1784 | 0.1784 | 0.1783 | 0.1655 |
| **2.6** | 0.1896 | 0.1861 | 0.1753 | 0.2281 | 0.2017 | 0.1753 | 0.1753 | 0.1752 | 0.1629 |
| **2.7** | 0.1861 | 0.1827 | 0.1723 | 0.2234 | 0.1979 | 0.1723 | 0.1723 | 0.1721 | 0.1603 |
| **2.8** | 0.1827 | 0.1793 | 0.1693 | 0.2189 | 0.1942 | 0.1693 | 0.1693 | 0.1692 | 0.1578 |
| **2.9** | 0.1794 | 0.1761 | 0.1665 | 0.2145 | 0.1906 | 0.1665 | 0.1665 | 0.1664 | 0.1553 |
| **3** | 0.1762 | 0.173 | 0.1637 | 0.2103 | 0.1871 | 0.1637 | 0.1637 | 0.1636 | 0.1529 |

Table 1**:** Interface velocity for different values of m, $M_1$, M2 and σ with fixed P= -1, t = 0.1, A = 0.1 and $\varepsilon$ = 0.1.

The variation of interface velocity is calculated from equation (52) for different values of the ratio of viscosity m, with effect of M1, $M_2$ and $\sigma$ and is shown in the Table. 1. It is found that the interface velocity decreases with the increment in the ratio of viscosity m, with the effect of magnetic field parameters $M_1$, $M_2$ and porous medium parameter $\sigma$ .For a given ratio of viscosity m, the interface velocity decreases with increasing of magnetic field $M_1$, $M_2$and porous medium parameter $\sigma$

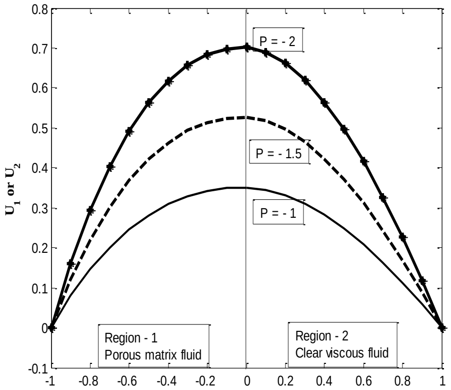

Fig. 2. Velocity profiles for different values of Pressure gradient P.

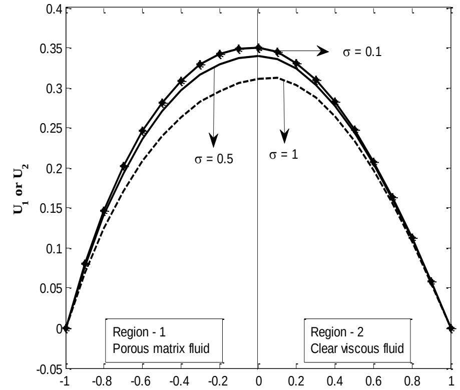

Fig. 3. Velocity profiles for different values of Porous mediam parameter σ.

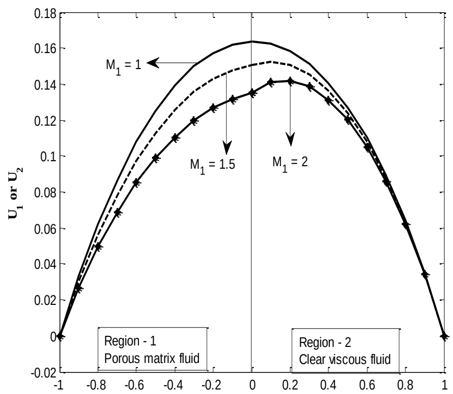

Fig. 4. Velocity profiles for different values of Magnetic field parameter $M_1$

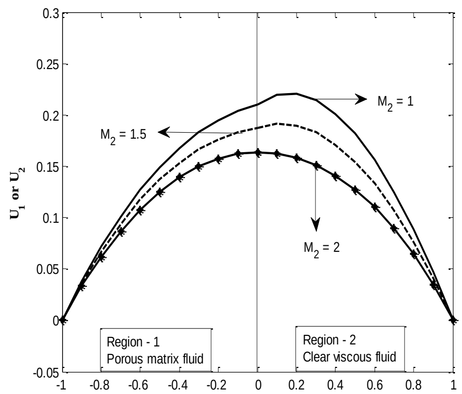

Fig. 5. Velocity profiles for different values of Magnetic field parameter $M_2$.

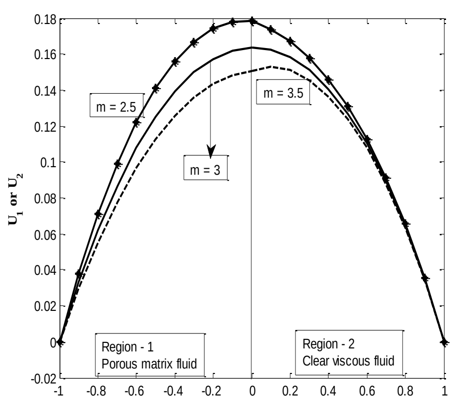

Fig. 6. Velocity profiles for different values of ratio of viscosity m .

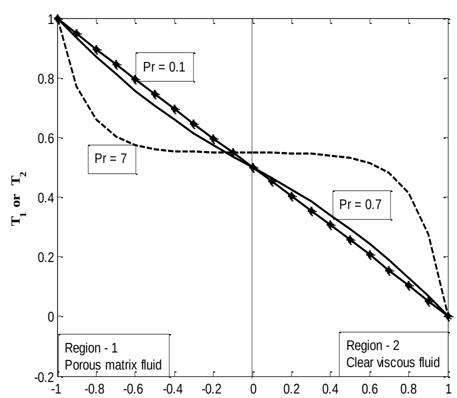

Fig. 7 Temperature profile for different values of Prandtl number Pr

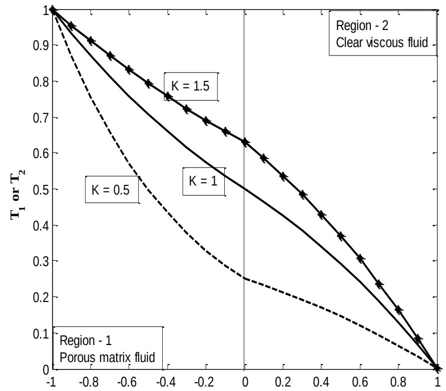


Fig. 8. Temperature profile for different values of thermal conductivities K .

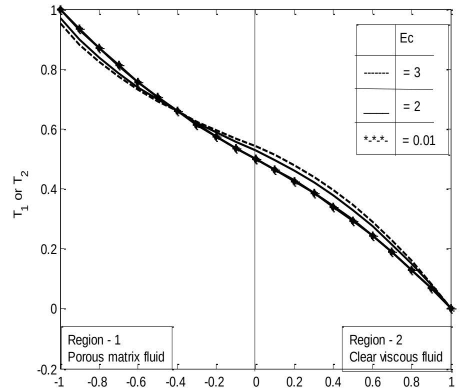


Fig. 9. Temperature profile for different values of Eckert number Ec

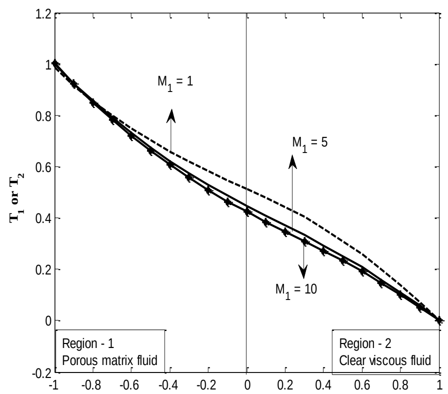


Fig. 10. Temperature profile for different values of Magnetic field parameter $M_1$

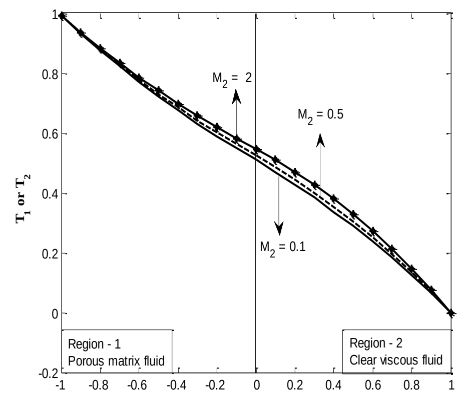


Fig. 11. Temperature profile for different values of Magnetic field parameter $M_2$.

## ACKNOWLEDGEMENTS

One of the Author Prof. S.Sreenadh expresses thanks to UGC for providing financial support through the Major Research Project to undertake this work.

## REFERENCES

[1] Brinkman, H. C., “A calculation of the viscous force exerted by a flowing fluid on a dense swarm of particles”, *Appl. Sci. Research*, A1; 27-34, 1947.

[2] Beavers, G. S., and Joseph, D. D., “Boundary conditions at a natural permeable wall” *J. Fluid Mechanics*, **30,** 197-207, 1967.

[3] Richardson, S., “A model for the boundary conditions of a porous media”, Part-II, *J. Fluid Mech.*, **49**, 319, 1971.

[4] Rudraiah, N., Ramanaiah, B. K., and Rajasekhara, B. M. “Hartmann flow over a

permeable bed", *Int. J. Engg. Sci*., **13**, 1-24, 1975.

[5] Rudraiah, N., Chandrasekhara, B. C., Veerabhadraiah, R., and Nagaraj, S.T., PGSA series, Bangalore University Publication, India. 1978.

[6] Rudraiah, N., and Wilfred, V., Nat. Acad. Sci. Letters., **3**, 49,1980.

[7] Vajravelu, K., Sreenadh, S. and Arunachalam, P. V., "Combined free and forced convection in an inclined channel with permeable boundaries", *Journal of Mathematical Analysis and Applications*, **166**, 393-403, 1992.

[8] Malashetty, M. S., and Leela, V., "Two fluid flow and heat transfer in an inclined channel containing porous and fluid layer", *J. Heat Mass Transfer*, **40**, 871- 876, 1992.

[9] Chamkha, A. J., "Flow of two-immiscible fluids in porous and non-porous channel", Transactions of ASME, *J. Fluid Engg*., **122**, 117-124, 2000.

[10] Malashetty, M. S., Umavathi, J. C. and Prathap kumar, J., "Convective flow and heat transfer in composite porous medium", *Journal of porous media*, **4,** 15-22, 2001.

[11] Khan, W. A., Culham, J. R. and Yovanovich, M. M., "Fluid flow and heat transfer in Power-law fluids across circular cylinders, Analytical study", ASME. J., **128**, 870-877, 2006.

[12] Umavathi, J. C., Chamkha, A. J., Mateen, A., Al-mudhaf, A., "Unsteady oscillatory flow and heat transfer in a horizontal composite porous medium channel", *Nonlinear Analysis: Modeling and Control*, **14**, 397–415, 2009.

[13] Vajravelu, K., Sreenadh, S. and Lakshminarayana, P., "The influence of heat transfer on peristaltic transport of a Jeffrey fluid in a vertical porous stratum", *Commun Nonlinear Sci. Numer. Simulat*, **16,** 3107–3125, 2011.

[14] Vasudev, C., Rajeswararao, U., prabhakararao, G. and Subba reddy, M. V., "MHD peristaltic flow of a Newtonian fluid through a porous medium in an asymmetric vertical channel with heat transfer", *International Journal of Science and Advanced Technology.*, **1**, 3, 2011.