INTERNATIONAL JOURNAL OF RESEARCH IN COMPUTER APPLICATIONS AND ROBOTICS

Maple as a laboratory tool in the study of RCL circuits

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Abstract

This paper explores the use of the computer algebra and symbolic computational software Maple as a valuable tool in the experimental study of RCL circuits. The traditional series RCL circuit is presented to show the usefulness of Maple software as a laboratory tool that can solve the circuit equations and more importantly display signals in a way analogous to the conventional oscilloscope (CRO). The effect of variation of the circuit components and input excitation conditions can be investigated fairly quickly and results displayed visually than what can be achieved in a typical practical laboratory.

Keywords: Electric circuit; RCL; Maple; laboratory; CAS; symbolic; damping; computation.

1. Introduction

An understanding of electric circuits is a critical component in university education for physicists and engineers. Solving electric circuit problems involves four main tasks [1,2]. First, the student has to formulate the equations representing the physical situation of interest. Secondly, the student has to symbolically solve those equations for the desired terms. The third task is fairly an easy one where the numerical values are plugged in and numerical evaluations done. The fourth task is to check the correctness of the initial equations, the solutions and the final numerical results. Most students fail to do the first step, that is, they face difficulties in stating a physical problem in mathematical terms. They equally lack the competence to do the symbolic manipulations necessary for solving the relevant equations. The study of electric circuits, particularly RCL, requires that the students be familiar with solving ordinary differential equations (ODEs) for them to understand the oscillatory behaviour of the circuits.

A number of published research articles show that many engineering and science students have difficulties in learning technical subjects because they lack sufficient competence in mathematics [1-8]. Powerful software programs for symbolic algebra computations, numerical calculations, plotting graphs and diagrams have been designed to help in the teaching and learning of mathematics for applied sciences and engineering. Among these, computer algebra systems (CAS) software such as Maple [9], Mathematica [10] and Mathcad [11] are the most common. CAS programs allow the focus to be on concepts and understanding principles rather than on large amount of routine mathematical manipulation. In addition, CAS allows more real-life scientific and engineering problems and this can greatly motivate learners [12-14]. In this study, we employ Maple 16 as a tool in the teaching and learning of RCL circuits. Maple is an interactive software for computer algebra that can assist both educators and students to overcome the obstacle of mathematical difficulties [9, 14-18]. The powerful symbolic computation

employed by Maple can be used such that the learner can concentrate on applying the principles of *setting* equations, instead of technical details of *solving* equations [19-22]. In addition, Maple has very outstanding graphics to enhance visualisations [9].

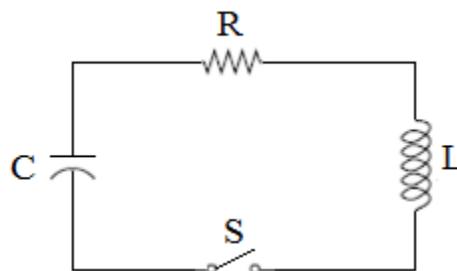## 2. Circuit theory and the ODES

Figure 1: Basic series RCL circuit

Consider the series RCL circuit in figure 1 where L is the inductance, C is the capacitance, R is resistance and S is the switch. The charge is represented by q [23, 24]. Applying Kirchhoff's rule and using the fact that current (i) is defined as the rate of flow of charge, $i = \dfrac{dq}{dt}$ and then the behaviour of the circuit in Figure 1 can be represented by:

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0 \tag{1}$$

Now consider an experimental situation where a function generator outputs a square wave of amplitude V_o with a frequency $f = 1/T$. Assuming the capacitor is fully charged from the previous cycle, then the initial conditions become $q(0) = CV_0$ and $\dot{q}(0) = 0$. The differential equation in (1) becomes [25-27]:

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = -V_0 \tag{2}$$

For the next cycle because the capacitor is negatively charged, we have the following initial conditions $q(1/T) = -CV_0$ and $\dot{q}(1/T) = 0$. The differential equation in (1) becomes:

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = V_0 \tag{3}$$

The differential equations can be solved and the performance of the circuit studied as a function of the circuit components and the frequency of the square wave input from the function generator. The output signal can be displayed on the CRO. Now Maple 16's superior graphics will be helpful in this regard. Equations (2) and (3) can now be solved and the results displayed. In a typical experiment, it is essential to study the effect of varying circuit component values on the output waveform in the context of oscillatory behaviour as defined by the three conditions [24,28,29]:

(a) Critical damping $\quad R^2 = \dfrac{4L}{C}$ \hfill (4)

(b) Under-damping $\quad R^2 < \dfrac{4L}{C}$ \hfill (5)

(c) Over-damping $\quad R^2 > \dfrac{4L}{C}$ \hfill (6)

In a typical laboratory setting, the student performing an experiment on the series RCL circuit can ideally fix the values of *C* and *L* and only vary *R*. Varying R can be achieved by using a resistor box to achieve the conditions stated in (a), (b) and (c). The resultant waveforms can then be displayed on the CRO. In this paper we demonstrate that the same experimental procedure can be achieved using Maple and the plots of the waveforms also done just as they would otherwise be when displayed on the CRO. Generally, a CRO is an indispensible instrument in the laboratory. The CRO can display voltage as a function of time, thus allowing experimental verification of a calculated quantity or result. The powerful graphics of Maple can perfectly simulate waveform displays analogous to the CRO traces. Performing the experiment in Maple will increase the student's understanding of the underlying theory since they can now directly observe the effect of varying component values on the output graphic waveform. Thus the ODEs will have physical significance, and will not just serve as mathematical constructs.

3. Procedure and Results

The ODEs defined in (2) and (3) can now be solved in Maple subject to the initial conditions stated. A typical experimental setup is when $R = 50\ \Omega$; $L = 10$ mH, $C = 0.1\mu$F, driven by a square wave signal with amplitude of 5 V and frequency of 200 Hz. The Maple worksheet in Section 3.1 shows the procedure for implementing the physical experiment for this set up [28]. Note that in Section 3.1 the commands in red represent the input into Maple while the blue shows the output. The screenshot showing the commands and the output waveform is shown in Figure 2.

3.1 Typical Maple Commands

```
> restart:
> V0:=5;T:=1/200;C:=0.1e-6;R:=50;L:=10e-3;
```

$$V0 := 5$$

$$T := \frac{1}{200}$$

$$C := 1.\,10^{-7}$$

$$R := 50$$

$$L := 0.010$$

```
> Eq1:=Q1(t)/C+diff(Q1(t),t)*R+L*diff(Q1(t),t$2)=-V0;
```

$$Eq1 := 1.\,10^{7}\,Q1(t) + 50\left(\frac{\mathrm{d}}{\mathrm{d}t}\,Q1(t)\right) + 0.010\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2}\,Q1(t)\right) = -5$$

```
> Soln1:=dsolve({Eq1,Q1(0)=C*V0,D(Q1)(0)=0},Q1(t));
```

$$Soln1 := Q1(t) = \frac{1}{159000000}\,e^{-2500\,t}\sin\!\left(2500\,\sqrt{159}\,t\right)\sqrt{159}$$

$$+ \frac{1}{1000000}\,e^{-2500\,t}\cos\!\left(2500\,\sqrt{159}\,t\right) - \frac{1}{2000000}$$

```
> assign(Soln1);
> Eq2:=Q2(t)/C+diff(Q2(t),t)*R+L*diff(Q2(t),t$2)=V0:
> Soln2:=dsolve({Eq2,Q2(T)=-C*V0,D(Q2)(T)=0},Q2(t)):
> assign(Soln2):
> Eq3:=Q3(t)/C+diff(Q3(t),t)*R+L*diff(Q3(t),t$2)=-V0:
> Soln3:=dsolve({Eq3,Q3(2*T)=C*V0,D(Q3)(2*T)=0},Q3(t)):
> assign(Soln3):
> with(plots):
> p1:=plot(Q1(t)/C,t=0..T):
> p2:=plot(Q2(t)/C,t=T..2*T):
> p3:=plot(Q3(t)/C,t=2*T..3*T):
> display([p1,p2,p3],axes=BOXED);
```
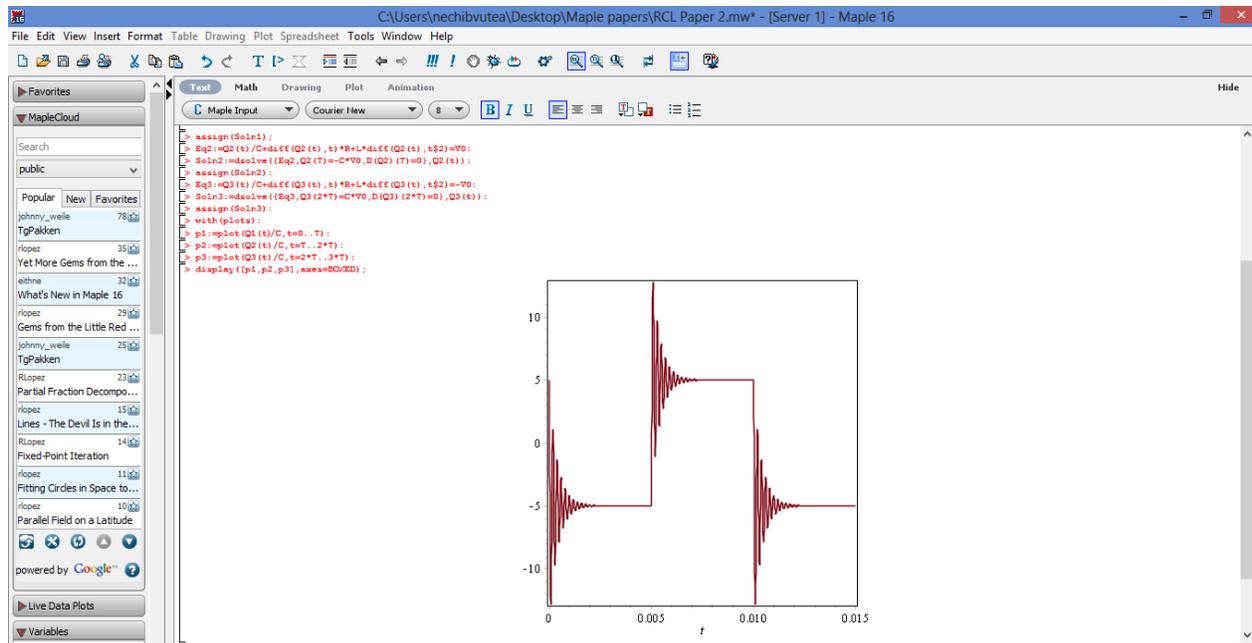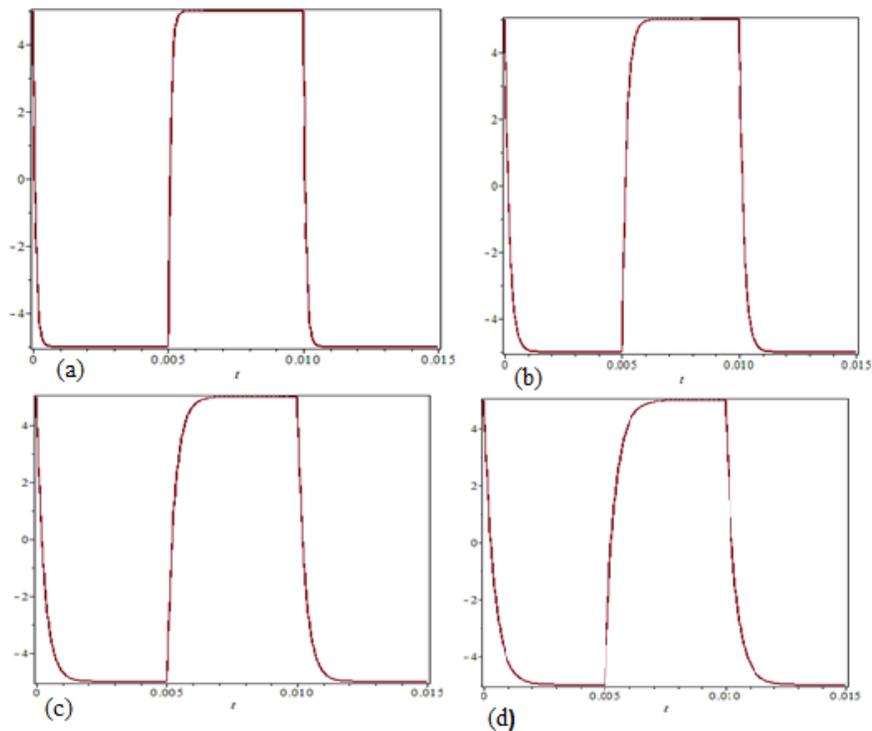
Figure 2: Screenshot of Maple worksheet

Using the procedure outlined in Section 3, critical damping, overdamping and underdamping could be successifully studied in Maple. The typical waveforms (equivalent to oscilloscope traces of voltage versus time) are shown in Figures 3-5.

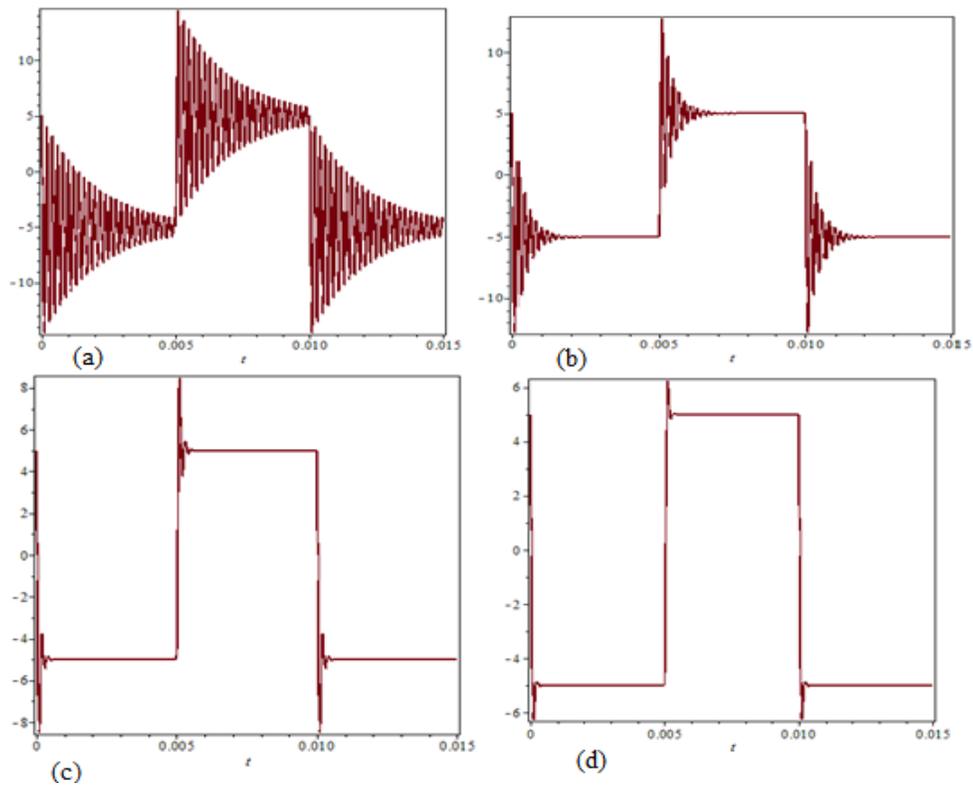Figure 3: Plots showing overdamping when $C = 0.1$ μF, $L = 10$ mH for different values of R [(a) 1 kΩ,(b) 2 kΩ (c) 3 kΩ, (d) 4 kΩ].

Figure 4: Plots showing underdamping when C = 0.1 µF, L =10 mH for different values of R [(a) 10Ω,(b) 250 Ω (c) 350 Ω, (d) 500 Ω].
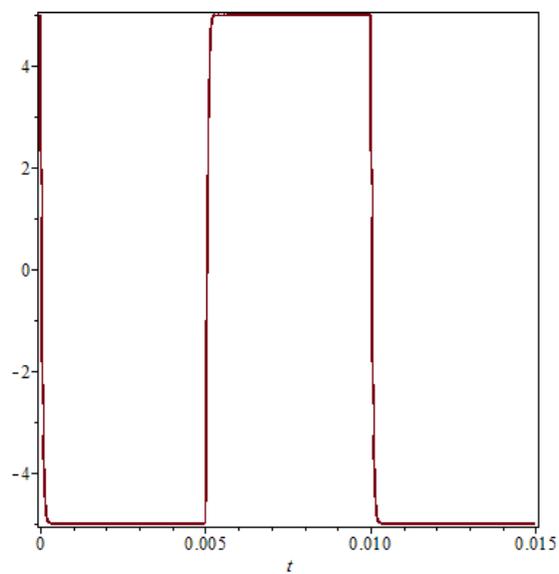

Figure 5: Plot showing critical damping when C = 0.1 µF, L =10 mH and R = 632.46 Ω

4. Discussion

We varied R and searched for a signal that has no undershoot and had the maximum decay rate. Typical CRO traces for over-damping, under-damping and critical damping are shown in Figures 3-5. From Figure 3, increasing R results in the increase in damping and the lagging edges of the traces are clipped. Figure 4 also shows the extent of under-damping as a function of increasing R. As R is increased, the ringing and overshooting reduces. For critical damping, the output waveform is a perfect replica of the square wave input signal.

Thus, this application example demonstrates the capabilities of Maple as an important tool that can assist students in understanding the mathematics and the physics behind the behaviour of RCL circuits. Once the student identifies the ODEs and the initial conditions, Maple solves these equations using the *dsolve command* and then the output waveforms can also be displayed mimicking the CRO display functions. The strength of employing Maple is that the student can quickly grasp the concepts involved, perform experimental study in the shortest possible time once Maple is mastered. In addition, the effect of varying circuit components on the damping can be easily explored compared to a practical laboratory setup where components have to be physically changed on a circuit board.

The electrical system can also be studied as an analogy of the mechanical oscillatory system where position (x) is equivalent to the charge (q), mass equivalent to inductance(L) damping coefficient (b) equivalent to resistance R, spring constant k equivalent to $1/C$ and force F equivalent to voltage V. This may aid the students' understanding of DC circuits in the context of mechanical systems.

5. Conclusion

In this work, we have presented the preliminary discussion of using maple as a laboratory tool in the study of RCL circuits. Maple can enhance the study of electric circuits and its powerful visual display facilities has the potential to greatly enhance the students' understanding of the principles involved, from solving ODEs to displaying the waveforms. In the future, we will extend the research to other linear and nonlinear circuits and solve them using Maple. These results will be used as undergraduate teaching material for Maple research and education.

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