INTERNATIONAL JOURNAL OF RESEARCH IN COMPUTER APPLICATIONS AND ROBOTICS
ISSN 2320-7345

# USING MAPLE TO EVALUATE THE DERIVATIVES OF SOME FUNCTIONS

**Chii-Huei Yu**

*Department of Management and Information, Nan Jeon Institute of Technology, Tainan City, Taiwan*
*chiihuei@mail.njtc.edu.tw*

## Abstract

This paper takes the mathematical software Maple as the auxiliary tool to study the differential problem of two types of functions. We can obtain the infinite series forms of any order derivatives of these two types of functions by using differentiation term by term and Leibniz differential rule, and hence greatly reduce the difficulty of evaluating their higher order derivative values. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. For this reason, Maple provides insights and guidance regarding problem-solving methods.
.

***Keywords***: Derivatives; infinite series forms; differentiation term by term; Leibniz differential rule; Maple.

## 1. Introduction

The computer algebra system (CAS) has been widely employed in mathematical and scientific studies. The rapid computations and the visually appealing graphical interface of the program render creative research possible. Maple possesses significance among mathematical calculation systems and can be considered a leading tool in the CAS field. The superiority of Maple lies in its simple instructions and ease of use, which enable beginners to learn the operating techniques in a short period. In addition, through the numerical and symbolic computations performed by Maple, the logic of thinking can be converted into a series of instructions. The computation results of Maple can be used to modify previous thinking directions, thereby forming direct and constructive feedback that can aid in improving understanding of problems and cultivating research interests. Inquiring through an online support system provided by Maple or browsing the Maple website (www.maplesoft.com) can facilitate further understanding of Maple and might provide unexpected insights. For the instructions and operations of Maple, we can refer to [1]-[7].

In calculus and engineering mathematics courses, determining the $n$-th order derivative value $f^{(n)}(c)$ of a function $f(x)$ at $x=c$, in general, needs to go through two procedures: firstly finding the $n$-th order derivative $f^{(n)}(x)$ of $f(x)$, and secondly taking $x=c$ into $f^{(n)}(x)$. These two procedures will make us face with increasingly complex calculations when calculating higher order derivative values of a function (i.e. $n$ is large), Therefore, to obtain the answers by manual calculations is not easy. In this paper, we mainly study the differential problem of the following two types of functions

$$f(x)=\frac{e^{a(p+1)x}\cos(p-1)(bx+c)-\lambda e^{apx}\cos p(bx+c)}{e^{2ax}-2\lambda e^{ax}\cos(bx+c)+\lambda^2} \tag{1}$$

$$g(x)=\frac{e^{a(p+1)x}\sin(p-1)(bx+c)-\lambda e^{apx}\sin p(bx+c)}{e^{2ax}-2\lambda e^{ax}\cos(bx+c)+\lambda^2} \tag{2}$$

, where $a,b,c,\lambda$ are real numbers, $p$ is an integer. We can obtain the infinite series forms of any order derivatives of these two types of functions mainly using differentiation term by term and Leibniz differential rule; these are the major results of this study (i.e., Theorems 1, 2), and hence greatly reduce the difficulty of evaluating their higher order derivative values. As for the related study of differential problems can refer to [8]-[16]. On the other hand, we propose two examples to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. For this reason, Maple provides insights and guidance regarding problem-solving methods.

## 2. Main Results

Firstly, we introduce some formulas used in this study.

***Euler's formula.***

$e^{iy}=\cos y+i\sin y$, where $y$ is any real number.

***DeMoivre's formula.***

$(\cos y+i\sin y)^n=\cos ny+i\sin ny$, where $n$ is any integer, $y$ is any real number.

***Geometric series.***

Suppose $z$ is a complex number, $|z|<1$. Then $\frac{1}{1-z}=\sum_{k=0}^{\infty}z^k$.

Next, we introduce two important theorems used in this paper.

***Differentiation term by term*** ([17]).

If, for all non-negative integer $k$, the functions $g_k:(a,b)\to R$ satisfy the following three conditions : (i) there exists a point $x_0\in(a,b)$ such that $\sum_{k=0}^{\infty}g_k(x_0)$ is convergent, (ii) all functions $g_k(x)$ are differentiable on open interval $(a,b)$, (iii) $\sum_{k=0}^{\infty}\frac{d}{dx}g_k(x)$ is uniformly convergent on $(a,b)$. Then $\sum_{k=0}^{\infty}g_k(x)$ is uniformly convergent and differentiable on $(a,b)$. Moreover, its derivative $\frac{d}{dx}\sum_{k=0}^{\infty}g_k(x)=\sum_{k=0}^{\infty}\frac{d}{dx}g_k(x)$.

***Leibniz differential rule*** ([18]):

Let $n$ be a positive integer, and $f(x),g(x)$ are functions such that their $m$-th order derivatives $f^{(m)}(x),g^{(m)}(x)$ exist for all $m=1,\dots,n$. Then the $n$-th order derivative of the product function $f(x)g(x)$,

$$(fg)^{(n)}(x)=\sum_{m=0}^{n}\binom{n}{m}f^{(n-m)}(x)g^{(m)}(x)$$

, where $\binom{n}{m} = \frac{n!}{m!(n-m)!}$.

Before deriving our main results, we need a lemma.

***Lemma.*** Suppose $z$ is a complex number, $p$ is any integer, $\lambda$ is a real number, $\lambda \neq 0$ and $|z| \neq |\lambda|$. Then

$$\frac{z^p}{z-\lambda} = -\sum_{k=0}^{\infty} \frac{1}{\lambda^{k+1}} z^{k+p} \quad \text{if } |z| < |\lambda| \tag{3}$$

$$= \sum_{k=0}^{\infty} \lambda^k z^{-k-1+p} \quad \text{if } |z| > |\lambda| \tag{4}$$

***Proof.*** If $|z| < |\lambda|$, then $\frac{z^p}{z-\lambda} = -z^p \cdot \frac{1}{\lambda} \cdot \frac{1}{1-\frac{z}{\lambda}}$

$$= -z^p \cdot \frac{1}{\lambda} \cdot \sum_{k=0}^{\infty} \left(\frac{z}{\lambda}\right)^k \quad \text{(By geometric series)}$$

$$= -\sum_{k=0}^{\infty} \frac{1}{\lambda^{k+1}} z^{k+p}$$

If $|z| > |\lambda|$, then $\frac{z^p}{z-\lambda} = z^p \cdot \frac{1}{z} \cdot \frac{1}{1-\frac{\lambda}{z}}$

$$= z^p \cdot \frac{1}{z} \cdot \sum_{k=0}^{\infty} \left(\frac{\lambda}{z}\right)^k \quad \text{(By geometric series)}$$

$$= \sum_{k=0}^{\infty} \lambda^k z^{-k-1+p} \qquad \blacksquare$$

The following is the first result in this study; we obtain the infinite series forms of any order derivatives of function (1).

***Theorem 1***. Let $a, b, c, \lambda$ be real numbers, $p$ be an integer, $n$ be a positive integer. If the domain of

$$f(x) = \frac{e^{a(p+1)x}\cos(p-1)(bx+c) - \lambda e^{apx}\cos p(bx+c)}{e^{2ax} - 2\lambda e^{ax}\cos(bx+c) + \lambda^2} \text{ is } \left\{x \in R \,\middle|\, e^{ax} \neq |\lambda|, e^{2ax} - 2\lambda e^{ax}\cos(bx+c) + \lambda^2 \neq 0\right\}.$$

Case (i): If $e^{ax} < |\lambda|$, then the $n$-th order derivative of $f(x)$,

$$f^{(n)}(x) = -\sum_{k=0}^{\infty} \frac{(k+p)^n}{\lambda^{k+1}} e^{a(k+p)x} \sum_{m=0}^{n} \binom{n}{m} a^{n-m} b^m \cdot \cos\left[(k+p)(bx+c) + \frac{m\pi}{2}\right] \tag{5}$$

For all $x$ satisfy $e^{ax} < |\lambda|$ and $e^{2ax} - 2\lambda e^{ax}\cos(bx+c) + \lambda^2 \neq 0$.

Case (ii): If $e^{ax} > |\lambda|$, then the $n$-th order derivative of $f(x)$,

$$f^{(n)}(x) = \sum_{k=0}^{\infty} \lambda^k (-k-1+p)^n e^{a(-k-1+p)x} \sum_{m=0}^{n} \binom{n}{m} a^{n-m} b^m \cdot \cos\left[(-k-1+p)(bx+c) + \frac{m\pi}{2}\right] \quad (6)$$

For all $x$ satisfy $e^{ax} > |\lambda|$ and $e^{2ax} - 2\lambda e^{ax}\cos(bx+c) + \lambda^2 \neq 0$.

***Proof***. Case (i): If $e^{ax} < |\lambda|$, taking $z = e^{ax} \cdot e^{i(bx+c)}$ into (3), we obtain

$$\frac{[e^{ax} \cdot e^{i(bx+c)}]^p}{e^{ax} \cdot e^{i(bx+c)} - \lambda} = -\sum_{k=0}^{\infty} \frac{1}{\lambda^{k+1}} [e^{ax} \cdot e^{i(bx+c)}]^{k+p}$$

$$\Rightarrow \frac{e^{apx} \cdot e^{ip(bx+c)}}{e^{ax} \cdot e^{i(bx+c)} - \lambda} = -\sum_{k=0}^{\infty} \frac{1}{\lambda^{k+1}} e^{a(k+p)x} \cdot e^{i(k+p)(bx+c)} \quad \text{(By DeMoivre's formula)}$$

$$\Rightarrow \frac{e^{apx} \cdot [\cos p(bx+c) + i\sin p(bx+c)]}{[e^{ax} \cdot \cos(bx+c) - \lambda] + ie^{ax} \cdot \sin(bx+c)} = -\sum_{k=0}^{\infty} \frac{1}{\lambda^{k+1}} e^{a(k+p)x} \cdot [\cos(k+p)(bx+c) + i\sin(k+p)(bx+c)]$$

(By Euler's formula) (7)

By the equal of the real parts of both sides of (7), we obtain

$$f(x) = \frac{e^{a(p+1)x}\cos(p-1)(bx+c) - \lambda e^{apx}\cos p(bx+c)}{e^{2ax} - 2\lambda e^{ax}\cos(bx+c) + \lambda^2} = -\sum_{k=0}^{\infty} \frac{1}{\lambda^{k+1}} e^{a(k+p)x} \cdot \cos(k+p)(bx+c) \quad (8)$$

Therefore, the $n$-th order derivative of $f(x)$,

$$f^{(n)}(x) = -\sum_{k=0}^{\infty} \frac{1}{\lambda^{k+1}} \frac{d^n}{dx^n} [e^{a(k+p)x} \cdot \cos(k+p)(bx+c)] \quad \text{(By differentiation term by term)}$$

$$= -\sum_{k=0}^{\infty} \frac{1}{\lambda^{k+1}} \sum_{m=0}^{n} \binom{n}{m} [e^{a(k+p)x}]^{(n-m)} \cdot [\cos(k+p)(bx+c)]^{(m)} \quad \text{(By Leibniz differential rule)}$$

$$= -\sum_{k=0}^{\infty} \frac{(k+p)^n}{\lambda^{k+1}} e^{a(k+p)x} \sum_{m=0}^{n} \binom{n}{m} a^{n-m} b^m \cdot \cos\left[(k+p)(bx+c) + \frac{m\pi}{2}\right]$$

For all $x$ satisfy $e^{ax} < |\lambda|$ and $e^{2ax} - 2\lambda e^{ax}\cos(bx+c) + \lambda^2 \neq 0$.

Case (ii): If $e^{ax} > |\lambda|$, taking $z = e^{ax} \cdot e^{i(bx+c)}$ into (4), we have

$$\frac{[e^{ax} \cdot e^{i(bx+c)}]^p}{e^{ax} \cdot e^{i(bx+c)} - \lambda} = \sum_{k=0}^{\infty} \lambda^k [e^{ax} \cdot e^{i(bx+c)}]^{-k-1+p}$$

$$\Rightarrow \frac{e^{apx} \cdot [\cos p(bx+c) + i\sin p(bx+c)]}{[e^{ax} \cdot \cos(bx+c) - \lambda] + ie^{ax} \cdot \sin(bx+c)}$$

$$= \sum_{k=0}^{\infty} \lambda^k e^{a(-k-1+p)x} \cdot [\cos(-k-1+p)(bx+c) + i\sin(-k-1+p)(bx+c)] \quad (9)$$

By the equal of the real parts of both sides of (9), we obtain

$$f(x) = \frac{e^{a(p+1)x}\cos(p-1)(bx+c) - \lambda e^{apx}\cos p(bx+c)}{e^{2ax} - 2\lambda e^{ax}\cos(bx+c) + \lambda^2} = \sum_{k=0}^{\infty} \lambda^k e^{a(-k-1+p)x} \cdot \cos(-k-1+p)(bx+c)$$

Vol.1 Issue.4, Pg: 23-31
July 2013

(10)

Thus,

$$f^{(n)}(x)$$

$$= \sum_{k=0}^{\infty} \lambda^k \frac{d^n}{dx^n}[e^{a(-k-1+p)x} \cdot \cos(-k-1+p)(bx+c)] \quad \text{(By differentiation term by term)}$$

$$= \sum_{k=0}^{\infty} \lambda^k \sum_{m=0}^{n} \binom{n}{m} [e^{a(-k-1+p)x}]^{(n-m)} \cdot [\cos(-k-1+p)(bx+c)]^{(m)} \quad \text{(By Leibniz differential rule)}$$

$$= \sum_{k=0}^{\infty} \lambda^k (-k-1+p)^n e^{a(-k-1+p)x} \sum_{m=0}^{n} \binom{n}{m} a^{n-m} b^m \cdot \cos\left[(-k-1+p)(bx+c) + \frac{m\pi}{2}\right]$$

For all $x$ satisfy $e^{ax} > |\lambda|$ and $e^{2ax} - 2\lambda e^{ax}\cos(bx+c) + \lambda^2 \neq 0$ ■

Next, we determine the infinite series forms of any order derivatives of function (2).

***Theorem 2***. If the assumptions are the same as Theorem 1, and suppose the domain of the function

$$g(x) = \frac{e^{a(p+1)x}\sin(p-1)(bx+c) - \lambda e^{apx}\sin p(bx+c)}{e^{2ax} - 2\lambda e^{ax}\cos(bx+c) + \lambda^2} \text{ is } \left\{x \in R \middle| e^{ax} \neq |\lambda|, e^{2ax} - 2\lambda e^{ax}\cos(bx+c) + \lambda^2 \neq 0\right\}.$$

Case (i): If $e^{ax} < |\lambda|$, then the $n$-th order derivative of $g(x)$,

$$g^{(n)}(x) = -\sum_{k=0}^{\infty} \frac{(k+p)^n}{\lambda^{k+1}} e^{a(k+p)x} \sum_{m=0}^{n} \binom{n}{m} a^{n-m} b^m \cdot \sin\left[(k+p)(bx+c) + \frac{m\pi}{2}\right] \tag{11}$$

For all $x$ satisfy $e^{ax} < |\lambda|$ and $e^{2ax} - 2\lambda e^{ax}\cos(bx+c) + \lambda^2 \neq 0$.

Case (ii): If $e^{ax} > |\lambda|$, then the $n$-th order derivative of $g(x)$,

$$g^{(n)}(x) = \sum_{k=0}^{\infty} \lambda^k (-k-1+p)^n e^{a(-k-1+p)x} \sum_{m=0}^{n} \binom{n}{m} a^{n-m} b^m \cdot \sin\left[(-k-1+p)(bx+c) + \frac{m\pi}{2}\right] \tag{12}$$

For all $x$ satisfy $e^{ax} > |\lambda|$ and $e^{2ax} - 2\lambda e^{ax}\cos(bx+c) + \lambda^2 \neq 0$.

***Proof***. Case (i): If $e^{ax} < |\lambda|$, then by the equal of the imaginary part of both sides of (7), we obtain

$$g(x) = \frac{e^{a(p+1)x}\sin(p-1)(bx+c) - \lambda e^{apx}\sin p(bx+c)}{e^{2ax} - 2\lambda e^{ax}\cos(bx+c) + \lambda^2} = -\sum_{k=0}^{\infty} \frac{1}{\lambda^{k+1}} e^{a(k+p)x} \cdot \sin(k+p)(bx+c) \tag{13}$$

Thus, the $n$-th order derivative of $g(x)$,

$$g^{(n)}(x) = -\sum_{k=0}^{\infty} \frac{1}{\lambda^{k+1}} \frac{d^n}{dx^n}[e^{a(k+p)x} \cdot \sin(k+p)(bx+c)] \quad \text{(By differentiation term by term)}$$

$$= -\sum_{k=0}^{\infty} \frac{1}{\lambda^{k+1}} \sum_{m=0}^{n} \binom{n}{m} [e^{a(k+p)x}]^{(n-m)} \cdot [\sin(k+p)(bx+c)]^{(m)} \quad \text{(By Leibniz differential rule)}$$

$$= -\sum_{k=0}^{\infty} \frac{(k+p)^n}{\lambda^{k+1}} e^{a(k+p)x} \sum_{m=0}^{n} \binom{n}{m} a^{n-m} b^m \cdot \sin\left[(k+p)(bx+c) + \frac{m\pi}{2}\right]$$

For all $x$ satisfy $e^{ax} < |\lambda|$ and $e^{2ax} - 2\lambda e^{ax}\cos(bx+c) + \lambda^2 \neq 0$.

Case (ii): If $e^{ax} > |\lambda|$, by the equal of the imaginary parts of both sides of (9), we obtain

$$g(x) = \frac{e^{a(p+1)x}\sin(p-1)(bx+c) - \lambda e^{apx}\sin p(bx+c)}{e^{2ax} - 2\lambda e^{ax}\cos(bx+c) + \lambda^2} = \sum_{k=0}^{\infty} \lambda^k e^{a(-k-1+p)x} \cdot \sin(-k-1+p)(bx+c) \tag{14}$$

Therefore,

$$g^{(n)}(x)$$

$$= \sum_{k=0}^{\infty} \lambda^k \frac{d^n}{dx^n}[e^{a(-k-1+p)x} \cdot \sin(-k-1+p)(bx+c)] \quad \text{(By differentiation term by term)}$$

$$= \sum_{k=0}^{\infty} \lambda^k \sum_{m=0}^{n} \binom{n}{m} [e^{a(-k-1+p)x}]^{(n-m)} \cdot [\sin(-k-1+p)(bx+c)]^{(m)} \quad \text{(By Leibniz differential rule)}$$

$$= \sum_{k=0}^{\infty} \lambda^k (-k-1+p)^n e^{a(-k-1+p)x} \sum_{m=0}^{n} \binom{n}{m} a^{n-m} b^m \cdot \sin\left[(-k-1+p)(bx+c) + \frac{m\pi}{2}\right]$$

For all $x$ satisfy $e^{ax} > |\lambda|$ and $e^{2ax} - 2\lambda e^{ax}\cos(bx+c) + \lambda^2 \neq 0$

## 3. Examples

Aimed at the differential problem of the two types of functions in this study, we provide two examples to determine the infinite series forms of their any order derivatives and evaluate some higher order derivative values practically. On the other hand, we use Maple to calculate the approximations of these higher order derivative values and their infinite series forms for verifying our answers.

***Example 1***. Suppose the domain of the function

$$f(x) = \frac{e^{-4x}\cos 4(3x-2) - 5e^{-6x}\cos 3(3x-2)}{e^{4x} - 10e^{2x}\cos(3x-2) + 25} \tag{15}$$

Is $\{x \in R \mid e^{2x} \neq 5, e^{4x} - 10e^{2x}\cos(3x-2) + 25 \neq 0\}$ (the case of $a = 2, b = 3, c = -2, p = -3, \lambda = 5$ in Theorem 1).

Case (1): If $e^{2x} < 5$, i.e., $x < \frac{1}{2}\ln 5 \cong 0.8047$.

By Case (i) of Theorem 1, we obtain any $n$ -th order derivative of $f(x)$,

$$f^{(n)}(x) = -\sum_{k=0}^{\infty} \frac{(k-3)^n}{5^{k+1}} e^{2(k-3)x} \sum_{m=0}^{n} \binom{n}{m} 2^{n-m} \cdot 3^m \cdot \cos\left[(k-3)(3x-2) + \frac{m\pi}{2}\right] \tag{16}$$

For all $x$ satisfy $e^{2x} < 5$ and $e^{4x} - 10e^{2x}\cos(3x-2) + 25 \neq 0$.

Thus, we can evaluate the 4-th order derivative value of $f(x)$ at $x = \frac{1}{3}$,

$$f^{(4)}\left(\frac{1}{3}\right) = -\sum_{k=0}^{\infty} \frac{(k-3)^4}{5^{k+1}} e^{2k/3-2} \sum_{m=0}^{4} \binom{4}{m} 2^{4-m} \cdot 3^m \cdot \cos\left(-k+3+\frac{m\pi}{2}\right) \qquad (17)$$

In the following, we use Maple to verify the correctness of (17).

```
>f:=x->(exp(-4*x)*cos(12*x-8)-5*exp(-6*x)*cos(9*x-6))/(exp(4*x)-10*exp(2*x)*cos(3*x-2)+25);
```

$$f := x \to \frac{e^{-4x}\cos(12x-8) - 5e^{-6x}\cos(9x-6)}{e^{4x} - 10e^{2x}\cos(3x-2) + 25}$$

```
>evalf ((D@@4)(f)(1/3),14);
```

-323.63861520538

```
>evalf(-sum((k-3)^4/5^(k+1)*exp(2*k/3-2)*sum(4!/(m!*(4-m)!)*2^(4-m)*3^m*cos(-k+3+m*Pi/2),m=0..4),k=0..infinity),14);
```

-323.63861520543

Case (2): If $e^{2x} > 5$, i.e., $x > \frac{1}{2}\ln 5 \cong 0.8047$.

By Case (ii) of Theorem 1, we can determine any $n$ -th order derivative of $f(x)$,

$$f^{(n)}(x) = \sum_{k=0}^{\infty} 5^k (-k-4)^n e^{2(-k-4)x} \sum_{m=0}^{n} \binom{n}{m} 2^{n-m} \cdot 3^m \cdot \cos\left[(-k-4)(3x-2) + \frac{m\pi}{2}\right] \qquad (18)$$

For all $x$ satisfy $e^{2x} > 5$ and $e^{4x} - 10e^{2x}\cos(3x-2) + 25 \neq 0$.

Hence, we obtain the 5-th order derivative value of $f(x)$ at $x = 1$,

$$f^{(5)}(1) = \sum_{k=0}^{\infty} 5^k (-k-4)^5 e^{-2k-8} \sum_{m=0}^{5} \binom{5}{m} 2^{5-m} \cdot 3^m \cdot \cos\left(-k-4+\frac{m\pi}{2}\right) \qquad (19)$$

We also use Maple to verify the correctness of (19).

```
>evalf ((D@@5)(f)(1),22);
```

10.48701240375211003658

```
>evalf(sum(5^k*(-k-4)^5*exp(-2*k-8)*sum(5!/(m!*(5-m)!)*2^(5-m)*3^m*cos(-k-4+m*Pi/2),m=0..5),k=0..infinity),22);
```

10.48701240375211004439

***Example 2***. Assume the domain of the function

$$g(x) = \frac{e^{15x}\sin(2x+1) + 4e^{10x}\sin 2(2x+1)}{e^{10x} + 8e^{5x}\cos(2x+1) + 16} \qquad (20)$$

Is $\left\{x \in R \middle| e^{5x} \neq 4, e^{10x} + 8e^{5x}\cos(2x+1) + 16 \neq 0\right\}$ (the case of $a = 5, b = 2, c = 1, p = 2, \lambda = -4$ in Theorem 2).

Case (1): If $e^{5x} < 4$, i.e., $x < \frac{1}{5}\ln 4 \cong 0.2772$.

By Case (i) of Theorem 2, we can evaluate any $n$ -th order derivative of $g(x)$,

$$g^{(n)}(x)=-\sum_{k=0}^{\infty}\frac{(k+2)^n}{(-4)^{k+1}}e^{5(k+2)x}\sum_{m=0}^{n}\binom{n}{m}5^{n-m}\cdot 2^m\cdot\sin\left[(k+2)(2x+1)+\frac{m\pi}{2}\right] \qquad (21)$$

For all $x$ satisfy $e^{5x}<4$ and $e^{10x}+8e^{5x}\cos(2x+1)+16\neq 0$.

Therefore, we can obtain the 6-th order derivative value of $g(x)$ at $x=-1$,

$$g^{(6)}(-1)=-\sum_{k=0}^{\infty}\frac{(k+2)^6}{(-4)^{k+1}}e^{-5k-10}\sum_{m=0}^{6}\binom{6}{m}5^{6-m}\cdot 2^m\cdot\sin\left(-k-2+\frac{m\pi}{2}\right) \qquad (22)$$

Using Maple to verify the correctness of (22) as follows:

```
>g:=x->(exp(15*x)*sin(2*x+1)+4*exp(10*x)*sin(4*x+2))/(exp(10*x)+8*exp(5*x)*cos(2*x+1)+16);
```

$$g:=x\to\frac{e^{15x}\sin(2x+1)+4e^{10x}\sin(4x+2)}{e^{10x}+8e^{5x}\cos(2x+1)+16}$$

```
>evalf ((D@@6)(g)(-1),14);
```

5.1678534432644

```
>evalf(-sum((k+2)^6/(-4)^(k+1)*exp(-5*k-10)*sum(6!/(m!*(6-m)!)*5^(6-m)*2^m*sin(-k-2+m*Pi/2),m=0..6),k=0..infinity),14);
```

5.1678534432643

Case (2): If $e^{5x}>4$, i.e., $x>\frac{1}{5}\ln 4\cong 0.2772$.

By Case (ii) of Theorem 2, we can evaluate any $n$ -th order derivative of $g(x)$,

$$g^{(n)}(x)=\sum_{k=0}^{\infty}(-4)^k(-k+1)^n e^{5(-k+1)x}\sum_{m=0}^{n}\binom{n}{m}5^{n-m}\cdot 2^m\cdot\sin\left[(-k+1)(2x+1)+\frac{m\pi}{2}\right] \qquad (23)$$

For all $x$ satisfy $e^{5x}>4$ and $e^{10x}+8e^{5x}\cos(2x+1)+16\neq 0$.

Thus, we obtain the 5-th order derivative value of $g(x)$ at $x=\frac{2}{5}$,

$$g^{(5)}\left(\frac{2}{5}\right)=\sum_{k=0}^{\infty}(-4)^k(-k+1)^5 e^{-2k+2}\sum_{m=0}^{5}\binom{5}{m}5^{5-m}\cdot 2^m\cdot\sin\left(-\frac{9k}{5}+\frac{9}{5}+\frac{m\pi}{2}\right) \qquad (24)$$

In the following, we use Maple to verify the correctness of (24).

```
>evalf ((D@@5)(g)(2/5),20);
```

$1.13317923606905354\cdot 10^5$

```
>evalf(sum((-4)^k*(-k+1)^5*exp(-2*k+2)*sum(5!/(m!*(5-m)!)*5^(5-m)*2^m*sin(-9*k/5+9/5+m*Pi/2),m=0..5),k=0..infinity),18);
```

$1.13317923606905351\cdot 10^5$

Vol.1 Issue.4, Pg: 23-31
July 2013

## 4. Conclusion

From the above discussion, we know the differentiation term by term and the Leibniz differential rule play significant roles in the theoretical inferences of this study. In fact, the applications of these two theorems are extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related applications. On the other hand, Maple also plays a vital assistive role in problem-solving. In the future, we will extend the research topic to other calculus and engineering mathematics problems and solve these problems by using Maple. These results will be used as teaching materials for Maple on education and research to enhance the connotations of calculus and engineering mathematics.

## References

[1] C. T. J. Dodson and E. A. Gonzalez, *Experiments in Mathematics Using Maple*, New York: Springer-Verlag, 1995.
[2] D. Richards, *Advanced Mathematical Methods with Maple*, New York: Cambridge University Press, 2002.
[3] M. L. Abell and J. P. Braselton, *Maple by Example*, 3rd ed., New York: Elsevier Academic Press, 2005.
[4] R. J. Stroeker and J. F. Kaashoek, *Discovering Mathematics with Maple: An Interactive Exploration for Mathematicians, Engineers and Econometricians*, Basel: Birkhauser Verlag, 1999.
[5] C. Tocci and S. G. Adams, *Applied Maple for Engineers and Scientists*, Boston: Artech House, 1996.
[6] J. S. Robertson, *Engineering Mathematics with Maple*, New York: McGraw-Hill, 1996.
[7] F. Garvan, *The Maple Book*, London: Chapman & Hall/CRC, 2001.
[8] C. -H. Yu, "Application of Maple on solving some differential problems," *Proceedings of IIE Asian Conference 2013*, National Taiwan University of Science and Technology, Taiwan, no. 0108, 2013.
[9] C. -H. Yu, "Application of Maple on the differential problem of hyperbolic functions," *Proceedings of International Conference on Safety & Security Management and Engineering Technology 2012*, WuFeng University, Taiwan, pp. 481-484, 2012.
[10] C.-H. Yu, "A study on some differential problems with Maple," *Proceedings of 6th IEEE/International Conference on Advanced Infocomm Technology*, National United University, Taiwan, no. 00291, 2013.
[11] C. -H. Yu, "Application of Maple: taking the differential problem of rational functions as an example," *Proceedings of 2012 Optoelectronics Communication Engineering Workshop*, National Kaohsiung University of Applied Sciences, Taiwan, pp. 271-274, 2012.
[12] C. -H. Yu, " The differential problem of two types of exponential functions, " *Journal of Nan Jeon*, vol. 16, D1-1~11.
[13] C. -H. Yu, "Application of Maple: taking the evaluation of higher order derivative values of some type of rational functions as an example," *Proceedings of 2012 Digital Life Technology Seminar*, National Yunlin University of Science and Technology, Taiwan, pp.150-153, 2012.
[14] C. -H. Yu, " A study on the differential problem of some trigonometric functions, " *Journal of Jen-Teh*, vol. 10, in press.
[15] C. -H. Yu, "The differential problem of four types of functions," *Journal of Kang-Ning*, vol. 14, in press.
[16] C. -H. Yu, " A study on the differential problems using Maple," *International Journal of Computer Science and Mobile Computing* , vol. 2, issue. 7, pp. 7-12, 2013.
[17] T. M. Apostol, *Mathematical Analysis*, 2nd ed., Boston: Addison-Wesley, p230, 1975.
[18] L. Flatto, *Advanced Calculus*, Baltimore: The Williams & Wilkins, p50, 1976.